SOCIEDAD, CULTURA Y COGNICIÓN: INTERCONEXIONES EN LA EDUCACIÓN MATEMÁTICA

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Resumen

En este artículo sostenemos que la producción de conocimiento científico es un producto que está continuamente ligado a la realidad social en tanto es un movimiento constructivo caracterizado por la sociodinámica de la cultura. Elaborado en forma de ensayo argumentativo, discute la constitución de procesos socioculturales que movilizan lectura, comprensión y explicación de la realidad vivida y validada por la sociedad humana. Fue con este entendimiento que realizamos nuestras indagaciones, discusiones y reflexiones sobre las posibilidades de establecer una Educación Matemática centrada en la interconexión de las matemáticas, la sociedad, la cognición y la cultura (MSCC). Con base en nuestras reflexiones, apuntamos indicadores para la enseñanza interactiva e integradora de las matemáticas, lo que aboga por el desarrollo de un educador matemático que actúe como mediador del aprendizaje operacionalizado a favor de la emancipación y transformación social de los estudiantes.
Palabras clave: Matemáticas. Educación Matemática. Sociedad. Cognición. Cultura

SOCIETY, CULTURE, AND COGNITION: INTERCONNECTIONS IN MATHEMATICS EDUCATION

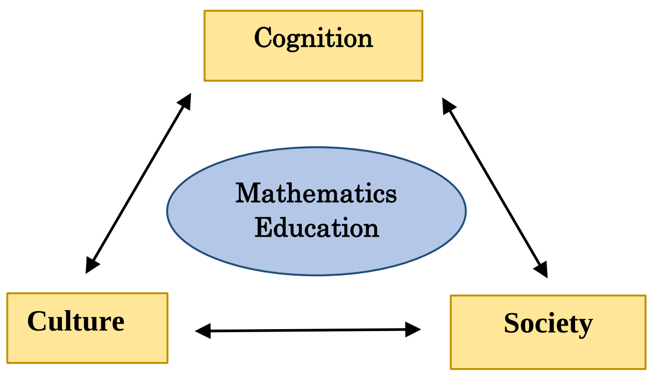
Abstract

In this article, we maintain that the production of scientific knowledge is a product that is continuously linked to social reality as it is a constructive movement characterized by the socio-dynamics of culture. Elaborated in the form of an argumentative essay, it discusses the constitution of socio-cultural processes that mobilize reading, understanding, and explanation of the reality experienced and validated by human society. It was with this understanding that we conducted our inquiries, discussions, and reflections on the possibilities of establishing a Mathematics Education centered on the interconnection of mathematics, society, cognition, and culture (MSCC). Based on our reflections, we point out indicators for interactive and integrative mathematics teaching, which argues for the development of a mathematical educator who acts as a mediator of operationalized learning in favor of the emancipation and social transformation of students.
Keywords: Mathematics, Mathematics Education, Society, Cognition, Culture.

PRELIMINARY CONSIDERATIONS

Society changes all the time. Education, being the sector of society responsible for disseminating the knowledge produced by the scientific community, has also been transformed throughout history. Although some teachers perceive the need for change, they often do not understand the urgency of effecting it. The production of scientific knowledge as a sociocultural product is not detached from this reality. The same dynamic occurs within mathematics. Therefore, we can state that mathematical production is subsidized by an interconnection that involves the society that produces it as thought and action (cognition), and as a culture, which subsidizes the organization and enunciation of rules for the installation of multiple sociocultural processes such as religions, arts, economy, politics, education, etc..., as shown in the infographic we see in Figure 1.

Figure 1



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This dynamic constitutes cognitive processes originated in the creative action of several mental operators driven by the need to read, understand, and explain the reality invented and validated by human society. Conducting my inquiries in this way, my objective in this article is to discuss how Mathematics Education can be constituted in the interconnection of society, culture, and cognition.

It is a strategy of betting on the pedagogical exploration of a possible interactive and integrative process based on the relationship between mathematics, society, cognition, and culture (MSCC) as proposed by Vergani (1991) with the aim of valuing this understanding of how the theme might be understood. To do that, this relationship must be considered as a central axis for the definition of what it means to be a mathematics educator and an agent of the mediation it effects, and thus be able to contribute to the accomplishment of emancipation and social transformation.

However, to build this movement effectively, it is necessary to reflect on important aspects implied by the triangular connection shown in Figure 1. First, it is prudent to reflect upon the nature of mathematics and its level of reality, which is supported by four fundamental pillars of Mathematics Education: the historical, anthropological, and foundational perspectives of mathematics application.

The first point to be considered regarding the bases of contextualization of mathematical production in the connection between society, cognition, and culture relies on history. Therefore, we need to think about how throughout the millennia human societies all over the world have organized themselves around the development of mathematics, so we can understand its current epistemological stage. Using this approach, some authors conceive mathematics education as a possibility of proposing an educational process based on principles and values advocated by mathematical culture, which means using mathematics as a vehicle, means, or instrument to educate society based on its developmental process.

In this context, some questions arise: how can we establish a historical reflection on society's millenary trajectory and its relationship with the development of mathematics alongside its path? How and why did different types of mathematics emerge at different historical moments? Does mathematics teaching need to consider these questions so that it is able to reflect on this movement? Does the mathematics we need today exhibit—or should it exhibit—the same representative features established five hundred years ago? Do we not need to add new approaches to mathematics such as those offered by computers, calculators, and digital technologies such as apps and software, for example, to meet the needs of society?

We do not intend to answer each of these questions in detail, but to present them as a provocative way of encouraging readers to undertake a reflective exercise of thinking about the situation and perhaps looking for new ways of reinventing themselves regarding the mathematics they study or teach. In order to analyze the situation historically, we need to study a little of its contextual configuration to identify how much human society has developed its scientific and technological production according to its interests and how these productions have been transformed according to its sociocultural needs. This is necessary because we consider that constructive moments cannot be discarded which were established according to the social needs of each historical moment.

It is a movement of construction and reconstruction which are both resignified at each historical moment and according to the requirements of social organizations configured in sociocultural activities, which are reformulated through processes of cognitive adaptability—a characteristic of human society.

We, as mathematics teachers, need to reasonably understand this historical perspective so that we can approach mathematics nowadays in a more comprehensive way when we reflect on how the ways of solving problems in the past have resulted in the knowledge we now have and how they can impact future knowledge. It means, therefore, that we should not take a step forward without looking at what happened in the past and thinking about what might happen further ahead. Towards this end, history can contribute a lot to the development of mathematics teaching based on a model of thinking focused on the social formation of those who learn.

Another aspect referring to the foundations of a mathematics education we can envision is the anthropological perspective, whose purpose is to understand how socio-cultural groups produce their mathematical practices and knowledge in their daily lives. Subsequently, the appropriation of production processes enables them to be mobilized and disseminated in the school environment.

In this case, we mean the possibilities of representing the diversities of mathematical ideas apprehended in different socio-cultural contexts to establish correlations between the various thought and practice strategies arising from these types of mathematics. As an example of these practices, we point out those that involve comparison, measurement, and quantification, among other mathematical skills and competencies. Thus, we create codes to establish a communication language so that everyone in the world understands mathematics.

Such a creation means building a grammar that constitutes an *ideographic mathematical writing*[1], if we admit that mathematics has a universalizing and historically universalized language over the centuries. This mathematical ideographic writing refers to the use of symbols that represent the meaning of what is expressed. The ideograms represent terms that incorporate mathematical meanings such as algarisms, equations, functions, graphs, etc., which all differ from the grammatical mathematical writing of each language, represented by the texts that enunciate mathematical situations.

[1] In this article, we interpret ideographic writing as a type of writing that represents language through ideograms or symbols that represent ideas and we reformulate our interpretation of this concept concerning mathematics from Jean (2008), Man (2002), Fischer (2009) and Levy (1997).

This mathematical ideographic writing goes through processes of creation, expansion, and space-time reformulation to explain new meanings as well as meanings attributed to mathematical objects that represent the mathematical practices and thoughts constituted in the physical world we inhabit and in the imagined world that we idealize.

Certainly, if we proceed in this way, we can conclude that the way of understanding mathematics disseminated in school education also has a characteristic of universality that derives from interrelations and intercommunications, domain and power. We can, therefore, also admit that in this cycle—which involves the triad society, cognition, and culture—the production of mathematics in the daily life of societies goes through processes of creation, resignification, and reconstruction, always depending on the socio-cultural needs of each social and cultural group at different historical moments.

How can we talk about and discuss mathematics education without mastering the mathematical knowledge historically produced, its limits and spatial-temporal discontinuities, and the modes of organization for introducing it into school systems? To act as mathematics educators, mathematics teachers need to take ownership of the mathematical notions that they will address in the classroom and of the different ways of approaching each topic, as well as the interactions that involve knowledge and the multiple contexts that make up a conceptual network of each subject to be addressed in the classroom. Otherwise, it will not be possible to implement mathematics education as intended.

In this sense, it is necessary to understand the historical-epistemological process of each mathematical theme, that is, the generative process of this knowledge, the reason for its organization and systematization. With this understanding, perhaps it is possible for the teachers to overcome the difficulties constantly encountered in the classroom when they are faced with students' questions about these mathematical reasons.

It means that, before thinking about becoming a good mathematics educator, teachers must first take ownership of the historical-conceptual development of the mathematical themes that will be addressed in their teaching activities. This aspect, therefore, constitutes another pillar or foundation that must be solidified towards the constitution of a grounded and useful mathematical education for the formation of a citizen-society.

There is another decisive perspective in this whole process, through which mathematics is also founded as a scientific culture. It is the perspective related to the applications of mathematics to the technological production of calculators, computers, internet networks, and information and communication technologies in general; these aspects are related to the production and use of instruments and vehicles suitable for the treatment of information. Often, these aspects were or still are disregarded or rejected by teachers, although they are present in our socio-historical and cultural context.

From that moment on, we can reflect on the integrative connections between the historical, anthropological, and foundational basis of mathematics and its applications so that we can discuss which of its characteristics are necessary for today's society. From these connections, perhaps we can establish guiding parameters for any given society's mathematical formation towards a possible future. We need to know how and why this type of mathematics would be needed today and what impact it will have on our tomorrow, deriving from that notion a historical and anthropological reflection on themes and relationships with the foundations of the mathematics it entails.

Likewise, we highlight how much the culture of today's society was inherited from a mathematical ancestry that was managed over the centuries through continuous socio-cultural dynamics in which the most varied types of technologies were used as supports to represent mathematical creations. It is due to this relevance that society admits the need for a socio-historical and cultural perspective of school mathematics, in which technological contributions favor the students' understanding of mathematical processes, in the interconnections that involve culture, technology, and society.

It is also important to reflect on three approaches through which human knowledge is presented in society: everyday knowledge, originating from cultural traditions; school knowledge, produced and disseminated by the school systems, and scientific knowledge, which reflects the proofs and demonstrations of mathematics validation in the scientific society. Thus, we can say that we still have a way to go in the school process, both in society as a whole and within our own culture, in search of understanding the integrative mobilizations related to everyday, scientific, and school knowledge, in which we find intrinsic relations between cultural tradition, science, and technique, as we will see in the following pages.

THE DEFINITION OF MATHEMATICS AMID CULTURAL TRADITION, SCIENCE, AND TECHNIQUE

What mathematics is, where does it come from and what is its nature? Even though these questions are as old as the objects of mathematical study – man, society, nature, and culture—

they continue to entice philosophers and mathematicians. We might ask: who else would care about this? Nevertheless, the presence of mathematical ideas in all domains of culture can prompt us, if not to answer, at least to understand the extent of these questions. This makes us want to know what mathematics is. This is an open and complex question.

Considered a necessary and indispensable knowledge to humanity, mathematics is an integral part of our life and, consequently, of any proposal for an educational curriculum. It has been established as a way of thinking about the world, but also as a scientific meta-discourse that we use to understand other areas of human knowledge. It is at once a way of reasoning for any individual and a knowledge tool used in several other fields of science. We can say that, largely, it is at the foundation of the resolution of the phenomena we find in nature, culture, and society and it is an answer to our need of codifying and explaining reality.

There is, therefore, a need for a reflection on the nature of mathematics, as well as on the general guidelines that permeate its teaching. Consequently, there is also a need of presenting didactic-methodological propositions that evidence the expansion of the ways of dealing with this scientific discipline in the field of mathematics education (cf. Mendes and Silva, 2018).

In this regard, we start from the principle that, throughout our social history, mathematics appears as an instrument that helps us to understand, describe, and modify social reality. It shows its utilitarian facet when it is used to communicate information produced by other scientific and technical areas. Let us take the example of how, in the past, the first numbering systems allowed us to count days and to understand climatic cycles, which contributed to increased agricultural production, was taken as a basis for the advancement of the physical sciences and, later, of biological, chemical, and social sciences too.

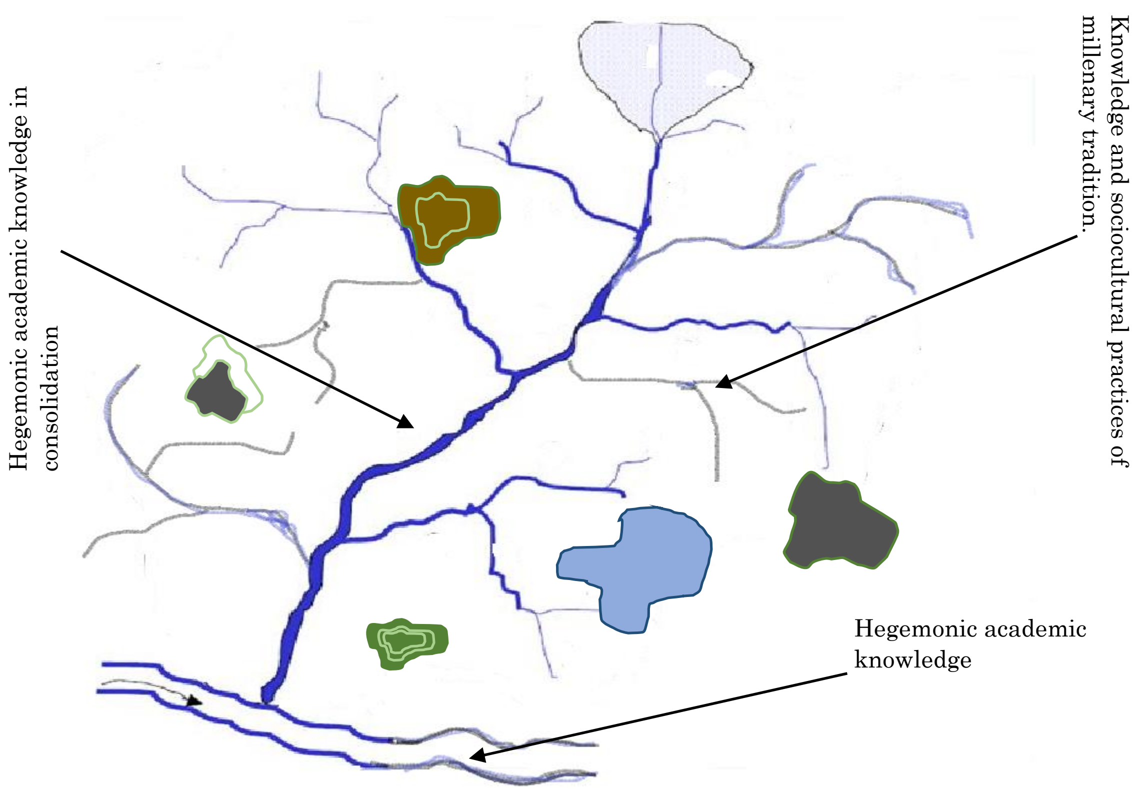
However, there are other ways of saying what mathematics and mathematical thinking are if we understand that mathematical science today is the exponential growth of pre-scientific knowledge that preceded it. In this regard, Ubiratan D'Ambrosio (2009) presents the idea of the hydrographic basin as a metaphor (Figure 2) to talk about mathematical thinking, and he states that:

> (...) we must recognize that the peripheral countries are confined to a situation of being no more than affluents of the main course of current scientific and technological development. This is the Metaphor of the Basin, which considers the knowledge of central countries as the mass of water of a large flow and the contribution of peripheral countries as the water of its tributaries. The waters of the large stream do not penetrate the affluents. Knowledge arrives at its destination, on the banks of affluent streams, after major transformations and

> it is, generally speaking, deficient. While the knowledge produced by the central nations runs its course like the great stream, the contribution of the affluent streams is, as a whole, trivial and marginal. Even so, the waters from the tributaries are incorporated into and give life to the great flow (D'Ambrosio, 2009, p. 16).

Thus, it makes sense to talk about ethnochemistry, ethnobiology, ethnomathematics, ethnophysics, etc. In this metaphor, different types of knowledge leave their sociocultural origins and integrate other types of knowledge in an interconnected way, as it happens to the waters of the different tributaries of a river in a hydrographic basin until they come together in a single bed that will carry the torrential waters of human socio-cultural knowledge. When they come together, these waters form a hybrid composed of the sediments carried by the different tributaries to the main bed. However, the waters of these tributaries will never return to their sources as they were when they migrated from their origins, here represented by cultural traditions. Rather, they will flow through new or old tributaries, but this time in the form of a new complex of multiple and interconnected knowledge.

Figure 2



Designed by the author to represent D'Ambrosio's metaphor

The basin metaphor presented by D'Ambrosio is therefore a more imaginative way of saying what mathematics is. It leads us to two reflections. The first one ensures us that we can define what a science is using a definition as well as an image. The second one emphasizes that the mathematics known to us today is largely a product of the early types of mathematics originated from diverse cultural traditions (cf. Lévi-Strauss, 1989).

Thus, using metaphors such as this one, we can think about mathematics as having three dimensions: as a science, as a game and as an art. The first one evokes a way of systematically and abstractly reading the world; the second one is expressed in the logical and playful aspect and the third one is characterized by its magical and imaginal form (cf. Mendes, 2003).

This is the meaning of D'Ambrosio's metaphor when he presents academic mathematics as the waters of the main riverbed in a hydrographic basin (Figure 2), one in which all the knowledge generated in the various specific sociocultural systems are tributaries of this river. Therefore, these other types of knowledge are considered ethno-knowledges, which are mixed and interconnected to generate a new knowledge (by combining its different types), in which the entire style and collective of thought and knowledge are represented, as Ludwik Fleck (1935; 2010) states when he discusses the genesis and development of a scientific fact.

About this process related to the genesis, development, and validation of the results presented, regarding the formulations of official mathematical theories, Bishop (1988) had already stated that throughout the 18th and 19th centuries several authoritative writers on mathematical history had presented illuminating explanations on such matters and had convincingly demonstrated that hegemonic mathematics has a cultural history.

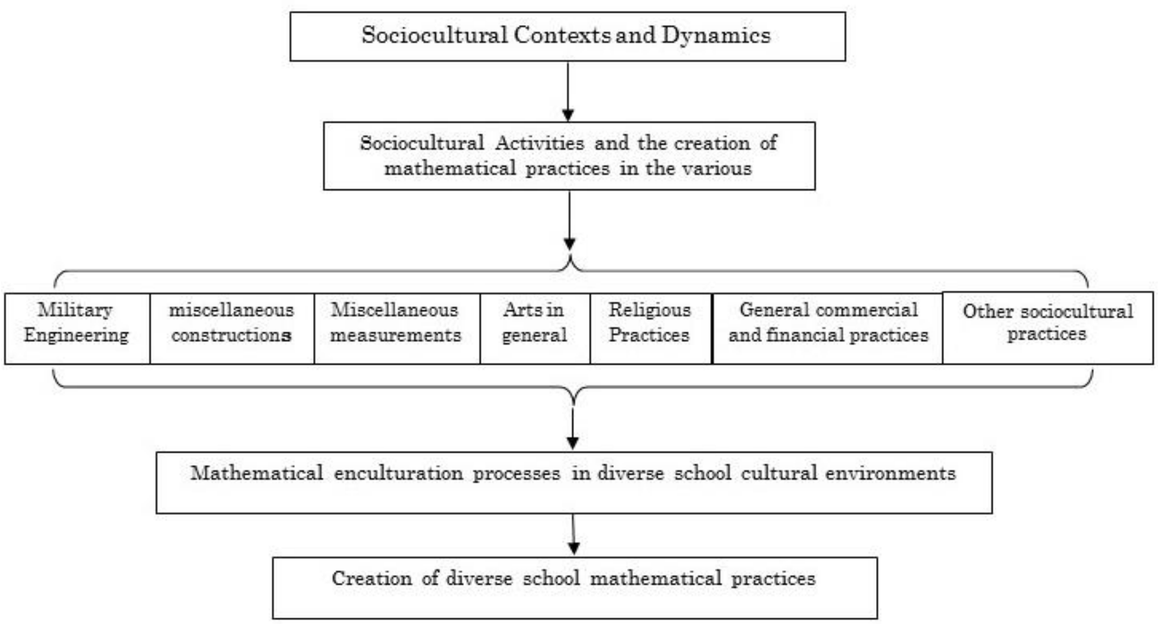
He explained which cultural history he was referring to by emphasizing that around the end of the 1980s in the 20th century evidence had emerged from anthropological and cross-cultural studies that not only supported the idea that mathematics has a cultural history, but also that from different histories emerged what can only be described as different mathematics (cf. Bishop, 1988).

From this perspective, the metaphor of the hydrographic basin alludes to the flow which turns multiple types of knowledge —originated in sociocultural practices of millenary tradition—into food or raw material that science transforms into what contemporaneity sees as domesticated, systematized knowledge, widely disseminated in a socio-cultural way. In this regard, in *The Savage Thought*, Claude Lévi-Strauss (1962; 1989), argues that it is from a science of the concrete (first science) that culture processes characterizations, purposes, and classifications of these first types of knowledge (*in natura* – natural types of knowledge) and reorganize forms of dissemination in the very society that produced it (domesticated knowledge).

> Wild thinking deepens its knowledge with the help of *imagines mundi*. It builds mental buildings that make it easier for it to understand the world to the extent that they resemble it. In this sense, it could be defined as analogical thinking. Yet in this sense it differs from domesticated thought—of which historical knowledge constitutes an aspect—[that is] the temporal order of a type of knowledge no longer discontinuous and analogical, but interstitial and uniting (Lévi-Strauss, 1989, p. 291).

As a science, mathematics is evidenced in the investigative activities which academic society has used to systematize the knowledge produced, practiced, and validated by the cultural tradition of each social group. It is a procedural movement of socio-cognitive investment aimed at the problematization, imagination, formulation, and representation of knowledge, with the objective of presenting solutions to problems and challenges that emerge in the socio-cultural context of each social group.

Figure 3



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The image in figure 3 reflects principles enunciated in theoretical-practical propositions presented and discussed by Abraham Moles (2007; 2012) and refers to the processes of scientific creation in the social dynamics of culture, which allows the possibility of transforming knowledge and practices according to social requirements and needs, as a kind of metamorphosis that involves socio-cultural knowledge and practices in continuous transformations intended to meet the inquiring demands of society in each time and place.

According to Moles (2007; 2012), it was through the social dynamics of cultures that processes of scientific creation were established in the scientific field, in which one of the ways of transferring thought systems from one field of knowledge to another was established as one of the most important and fruitful among the heuristic methods in the production of knowledge.

Thus, it is possible to understand how the adaptation of principles and methods from one field has been historically mobilized for the creation of new fields or subfields, such as in the numerical field, and the creative processes of the fields of numbers —whole, rational, real, and complex numbers—, as well as in the field of Euclidean and analytical geometry, or in the field of plane and spherical trigonometry.

In this sense, in a study on the real mechanisms of scientific creation, Moles (2007) asserts that there is almost an opposition between a science that is undergoing a transformative creative process and another one that is characterized as a finished, concluded science, contained in the announced results. The first one presents itself as a *science in construction* based on a network of evidence, not yet formalized, whose progress can almost always promote the emergence of the second one, called *formalized science*, which is condensed in the publications of articles, books, and research reports, which are usually subjected to questioning, discussions, and debates.

They are, therefore, two distinct ways of promoting the formalization of types of knowledge and practices towards scientific creation, through the social dynamics of culture, considering that

> the dynamic laws of thought are absolutely not the laws of formal logic, but rather a dynamic of scientific thought which involves creative activity, a scientific subconscious, a priori judgments, and real facts that represent the subconscious as a kind of reservoir of concepts, with later is charged with seeing how the emotional reservoir was filled (Moles, 2007, p. 52-53).

In a similar approach, but with an educational focus, Alan Bishop (1999) discussed relationships concerning the processes of mathematical enculturation established in cultural socio-dynamics, and proposes, from a sociocultural perspective, that we reflect on how do the theories, methods, and techniques that are continually mobilized for the teaching of mathematics emerge. In this sense, Bishop (1999) asserts that if we consider the social aspects of mathematics education, we will discern five important levels: cultural, social, institutional, pedagogical, and individual.

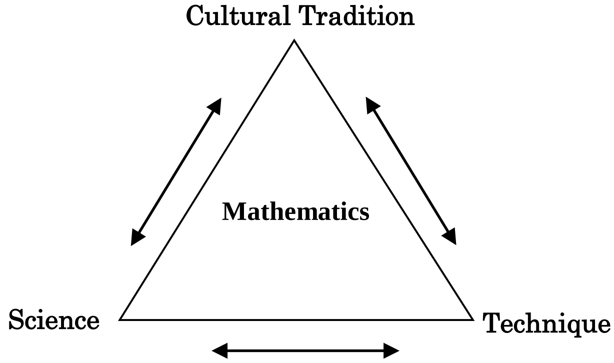
> The broader social group is the cultural one, and types of mathematics, being a cultural phenomenon, has a clearly supra-social nature. The different types of Mathematics are used in all societies and are the only school culture taught in most schools around the world. Furthermore, the rapid growth of the international community dedicated to mathematics education is an example of the supra-social condition of this school culture (Bishop, 1999, p. 32).

Thus, we reinforce the understanding that the first mathematical notions are interpreted as a cultural product generated in social activities and are related to the contexts that stimulate the creation of mathematical concepts and values underlying the mathematics they generate, thus explaining the procedural genesis of mathematical ideas, as highlighted by Bishop (1999).

An example of this metamorphosis of the cultural tradition related to mathematical knowledge and practices in mathematical science can be pointed out in studies involving the measurement of an object's shadow and give rise to explanations about knowledge such as trigonometric concepts. For many centuries, humanity relied on the shadows cast by objects under the sunlight to measure time, thus creating the first calendars: hours, days, lunar cycles, months, seasons, etc. (Mendes, 2022).

These interpretations of natural phenomena resulted in the determination of dates for the observance of religious practices and other socio-cultural activities, which led humanity towards mastering the counting of chronological time, generating a cycle of construction of the time recording machine, from the observation of the "sun streaks" to the crystallization of shadows through digital clocks. This is an example of how we can highlight the basis for the triadic relationship which places mathematics amid cultural traditions, science, and technique (Mendes, 2022).

Figure 4



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This relationship highlights another correlation: between the individual and their culture, which results in continued conflicts throughout the millennia, and occasioning the establishment of two distinct cultures, the human one and the technological one. This process represents the transformation of tradition and the emergence of different sciences and techniques. According to Davis and Hersh (1995),

> Being a human activity, Mathematics embodies all four components [nationalism, regionalism, freedom of the individual, and security]. It has much to gain from individual genius, but its development depends on the tacit approval of the community. As a major art form, it is humanistic; it is scientific-technological in its applications. To understand how and where mathematics and the human condition fit together, it is important to pay attention to all these components (Davis and Hersh, 1995, p. 70).

Such components lead us to reflect on the Bruter's statement (2000), according to which societies that display little technical and commercial evolution have recognized the pragmatic value of mathematics and made the necessary efforts to disseminate the knowledge that is necessary to meet the economic demands of each time. Thus, traditions were reinvented, and the sciences were constituted as a field of processing and validating results and conclusions that arise from mathematical practices originated in the most varied socio-cultural contexts.

Other dimensions often attributed to mathematics, such as gaming and the arts, manifest the transdisciplinary character of mathematics due to the cognitive aspects that establish mathematical thinking (cf. D'Ambrosio, 1997). This transdisciplinary nature emphasizes the importance of history and culture in the production of mathematical ideas and characterizes the place of mathematics in everyday, academic, and scientific contexts as the target of constant discussions that take place in academic circles. However, we are always worried about an aggravating factor that is always present in these discussions: a lack of definition for mathematics itself. In a way, this indefinability translates the open, broad character of this specific way of understanding knowledge, which has grown and developed in the most diverse directions throughout history.

In some dictionaries, we find mathematics as a science that investigates relationships between abstractly and logically defined entities; as a science of quantities and shapes to the extent that they are calculable and measurable. Or still: as a science that deals with relations and symbolism of numbers and quantities and that includes quantitative operations and solutions for quantitative problems. However, in such definitions we can highlight that mathematics itself is always described as an aspect that is present in the social environment, although unrelated to the people who produce it. Moreover, in each of the definitions there are essential elements that constitute guiding threads in the conceptions adopted for each way of approaching mathematics.

The gaming dimension attributed to mathematics is continuously manifested when we value the distinctive heuristics that is found in mathematical activity. This dimension is characterized by the emphasis placed on the value of a person's mathematical curiosity and imagination, considering the pleasure of handling information to achieve certain results (cf. Mendes, 2003).

The artistic dimension has been manifested throughout the historical development of mathematics. It is related to the creative spirit and the ability to accomplish the ideas generated through mathematical imagination. The artistic character is present, for example, in the mathematical models elaborated and applied to the geometry of ornaments— very common in architectural works, ceramic decorations, and handcrafted works such as tapestries, lace, Islamic arts etc.

Regarding the connections between science and technology, it is worth noticing the emphasis placed on them by Werner Heisenberg (2009). The author discusses human responsibility, concerning how the human position has changed in the relationship between nature and culture and recognizes that science and technique also became—especially from the 20th century onwards—responsible for these transformations. In this sense, the author points out that:

> (...) the diffusion of natural science and technology clearly shows how the nomological connections of the material world gain strength in reality. (...) We must, of course, be content at first with the connections by which our lives are

> amalgamated and supported, so it is certainly true that there must be very different regions or layers of reality (Heisenberg, 2009, p. 7).

These very different layers of reality mentioned above refer to the different regions of the chaotic reality in which knowledge is socially produced in each socio-cultural context at different times. In this social chaos where knowledge is produced, mathematics manifests its nature in each situation that gives rise to challenges, needs, and questions that promote empiricist, imaginative (intuitionist), and descriptive (logical and formalist) manifestations that determine ways of knowing and explaining through the facets of mathematics.

In an epistemological perspective that is similar to the aspects mentioned above, I have identified that Philip Kitcher (1984), in the book The Nature of Mathematical Knowledge, defends an empiricist approach regarding the development of mathematical knowledge in an a priori conception of mathematics, based on manipulations of the physical reality. The author argues that starting from the beginnings of practical explorations the production of mathematical knowledge has historically unfolded in the form of successive modifications that have involved and recreated a diversity of mathematical practices, stimulated by the presence of unsolved problems in the fields of society, science, and culture in general.

Kitcher (1984) states that this unfolding is generally considered an illustrated process if we investigate the most varied explanatory models on the historical-epistemological development of mathematics up to the present time. I would add to that the contributions of the variants arising from socio-cultural activities, which throughout history have provoked the expansion of readings, interpretations, and mathematical methods for understanding and explaining socially problematized situations.

Concerning the variants arising from socio-cultural activities, understood as ways of bringing about a system that would express the nature of mathematical knowledge, Teresa Vergani (1991) highlights that:

> In addition to being a precious inventory of a primordial aspect of human nature, the different dynamics of mathematics manifested in socio-cultural practices provide us with particularly objective indicators of situational analysis. (...) Hence mathematics, despite being diversified in different "accuracy" universes, can be looked at as potential instrument of hermeneutics in the field of socio-cognitive alterities that is particularly safe (Vergani, 1991, p. 22).

Therefore, the correlational exploration of these socio-cultural variants of past and present mathematizing activities can contribute to preventing the educational system from strengthening the threat of the aging or death of the mathematics taught in schools. This is because they all represent ways of organizing human thought to adopt mathematics as a strategy to read, understand, and explain socio-cultural reality.

MATHEMATICS AS A STRATEGY TO READ, UNDERSTAND AND EXPLAIN REALITY

The position encountered in the works of Zoltan Paul Dienes (1974) permeates a theoretical and practical conception in which mathematics does not mean only the discussion of a set of techniques, even if these are essential for the effective use of this form of cognitive expression. According to him, mathematics should be understood mainly as a structure of relationships in which there are effective connections between concepts linked to the idea of number, while its applications to problems involving such ideas are implemented in real life situations. This way of understanding mathematics seems to us to emphasize its constructive character as a thinking strategy to read, understand, and explain reality.

There are two other characteristic aspects that must be discussed due to their historical and socio-cultural importance in the construction of mathematics as knowledge: techniques and symbolism. These are two of its characteristic aspects which are extremely necessary for mathematical communication from one person to another and they constitute a kind of fundamental instrument of a language specially invented for this purpose.

Regarding the relationship between mathematical reality, symbolism, and technique, Nicolas Bouleau (2002) asserts that:

> The imaginary-real-symbolic Lacanian trilogy is convenient to support discussion. The imaginary is strongly distinguished from the symbolic by the fact that the image is not in itself an element of language whereas symbols, as soon as they are interpreted based on a given reality, express—by their combinations— paths not necessarily visited. We focus our attention above all on the relationship between that which is symbolic and that which is real: how can notation algebra demand the appearance of a new reality? The case of mathematics is reinforced here by that of physics, particularly during the genesis of quantum mechanics (p. 12-13).

Therefore, we can admit that mathematics is a language wrapped in significant elements whose objective is to express the meanings evidenced in each relationship that we structure to communicate our ideas. For Teresa Vergani (1993), this movement occurs through a mathematizing process, that is, a constructive procedure that includes the discovery of patterns and the search for some form of generalization of such discovered patterns. This process, however, demands from us some skills such as observation, tact, questioning, manipulation, experimentation, doubts, validation, and demonstration. This way of looking at mathematics as a living knowledge whose language is universalizing means that the creative imagination is integrated with logical rigor to generate and transmit the thought and meaning of those who use it.

Returning to Vergani's considerations (1993, p. 11), we can admit that mathematics is a discipline that is simultaneously abstract and concrete, rational and symbolic, pragmatic and playful. This means that its universalizing character lies in the balance between two fundamental extremes: creative imagination and logical rigor. Courant & Robbins (2000) also point out these antithetical forces that seek to characterize mathematics and define it as an expression of the human mind which reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are the following: logic and intuition, analysis and construction, generality and individuality.

Therefore, we see that the conceptions about mathematics as a cognitive expression always seek to establish a dialogue between action and reflection, part and whole. However, there must be a greater reflection on our part to establish links between knowledge and action in everyday and scientific situations, everyday situations and school situations, etc. These connections cause mathematics to be seen as a type of human and living knowledge. If we consider that, "everything that is alive is controversial" (Vergani, 1993, p. 17), we need to obtain a convergent path for these controversies of living mathematics. This path emerges as we perceive the existence of three correlational aspects in the type of mathematics that is socially produced and disseminated: the everyday, academic, and scientific aspects.

However, it is convenient to create a dialogue between these aspects so that we can approach mathematical knowledge from a transdisciplinary approach. The basis for such an approach can be sought in studies and research in the history of mathematics if we consider it as a unifying principle of these aspects. Thus, we will show how the production of mathematical knowledge occurred in different historical socio-cultural contexts to solve problems that arise daily in these spaces.

Regarding the first aspect, Carmen Gómez-Granell (1998, p. 19) states "everyday thinking is the result of direct social experience and is acquired through participation in the usual cultural practices of a given society". It means that everyday mathematical knowledge presents characteristics of both school-level and scientific knowledge. Thus, we admit that the everyday character of mathematics is directly related to its constructive processes in the socio-cultural context. In the same way, it also means that this aspect plays a fundamental role in the comprehension of reality and in people's actions in specific contexts of their activities.

For example, the technique the Egyptians used to determine the measurement of a pyramid's height by measuring their shadow shows that everyday mathematical activities can be taken as subsidies that influence the elaboration of school-level and scientific mathematics (geometry and trigonometry, for example). The Egyptians and Greeks, each with their own customs, political, economic, and social issues, in addition to their own mathematics, transformed a measurement technique perfecting it due to the climatic differences existing in each season of the year in those regions (cf. Mendes, 2022). Thus, they were able to achieve a more refined way of measuring distances, as well as the height of different objects around them.

In cases such as these, useful alternatives emerged for the development of mechanisms for measuring distances that until then had been considered inaccessible, such as the determination of the earth's radius, of the earth-moon distance, which caused the growth of navigation, among other decisive facts that transformed humanity.

After presenting these considerations, we can say that we disagree with scholars who consider this form of mathematical expression synonymous with an "inappropriate", "marginal", "popular", etc. kind of knowledge. Our position is justified by the fact that "everyday knowledge seems to represent human cognition better than formal knowledge. One cannot infer the conclusion that everyday reasoning is 'irrational' because it sometimes does not conform to the laws of logic; it only corresponds to another type of 'rationality', of a more pragmatic character" (Gómez-Granell, 1998, p. 16-17).

In this regard, we can exemplify the mathematical strategies developed by well-defined socio-cultural groups that investigate and use their local knowledge as a link between knowing and doing, so as to mediate how the students appropriate the school-level knowledge of the contexts in which such investigations occur.

Daily knowledge, therefore, is implicit, intuitive, usually arising from needs raised in the socio-cultural context, and plays an important role in the organization of school-level and scientific knowledge. In this sense, the history of mathematics is full of examples that show how the invention of new linguistic codes and more abstract symbols later contributed to the definition of what mathematics should or should not be taught in schools.

As for the school aspect of mathematics, we consider that it is directly connected to the organization of this knowledge, aiming at its socialization and dissemination. Although everyday mathematics is produced and usually consumed in the socio-cultural contexts in which it is generated, its officialization and diffusion happen through the school. This process takes place in a continuous dynamic that is characteristic of social interaction. This type of approach given to school-level mathematics should dialogue with everyday mathematics in a compatible and explicit way, relying much more on that which is implicit – the mathematical strategy of thought.

School-level mathematics should value everyday knowledge as a cognitive basis on which students can deepen their mathematical thinking until they organize it as school knowledge. This process must enrich the student's life through the formalization of mathematical ideas generated during the construction of models based on their experiences. We can see, therefore, that there is an urgent need to establish a dialogue between the three aspects of mathematical knowledge, considering the pedagogical value of this dialogue. In this sense, we must not forget that

> (...) the school is the institution in charge of transmitting scientific knowledge, but school knowledge is not scientific knowledge. Schools make adaptations that transform 'scientific knowledge' into 'knowledge that is taught'. ... This requires the elaboration of an epistemology of school knowledge, equal to the existing epistemology of scientific knowledge and everyday knowledge (Gómez-Granell 1998, p. 23).

We know in advance that the process of acquiring formal mathematical knowledge and language generally happens through schooling and intentional instruction. What we forget or do not value in this process are the intuitive components, typical of everyday thinking. They play an essential role in the construction of mathematical knowledge in the everyday perspective, as well as in the academic and scientific perspectives. If such components are not well used during the school learning process, there is a risk of presenting students with a mechanical, cold, static knowledge without any vital constitution.

To show our argument concerning this dialogue in a concrete way, we will present an example in the teaching of trigonometry: basic notions about proportionality as a prerequisite for learning trigonometric ratios. To reach our goals, we explored experimentally everyday aspects related to distance measurement practices and the ratio of similarities, so that students could draw conclusions about their experiences and eventually complete a process of formal elaboration of these conclusions. The experiment consisted of exploring measurements of the height of objects from the observation of the shadows they cast throughout the day; the assembly and operation of sundials; the measurement of height and distances between objects (cf. Mendes, 2022).

Regarding the scientific aspect of mathematics, we can say it is evident in the investigative activities that academic society has used to systematize everyday knowledge. This means that traditional knowledge, conceived to fulfill the needs of social groups, constitutes a network of information that will flow into technological production. Gómez-Granell (1998, p. 19) states that “the acquisition of scientific knowledge involves learning a method, a form of discourse that is not natural and demands a conscious and systematic effort towards clarification and rationalization.” It is important, however, to remember that when we begin to differentiate “theories” from “evidence”, we are building and appropriating a thought that is understood as scientific thought.

In this sense, we can give as an example the fact that, for many centuries, humans have used the shadow cast by an object under the sunlight to quantify, measure, and orient themselves according to time. The first time-oriented calendars emerged from this practice: hours, days, lunar cycles, months, seasons, etc. This was a step towards the ordering of religious practices and other social activities, guiding humanity towards the mastery of chronological time counting, generating a cycle of construction of the time recording machine, from the observation of "sun streaks" to the crystallization of shadows and digital clocks.

When we look at mathematics as a science, we place individual cognitive processes in relation to interactive processes of a social and cultural nature. Thus, it is possible to establish certain relationships between everyday aspects and school-level aspects pertaining to this knowledge to prompt, especially with mathematics educators, a reflective process regarding the frequent attempt to annul everyday mathematics using school-level mathematics. It is necessary, then, to reformulate our conceptions about what mathematical knowledge is from a scientific perspective. This reformulation permeates especially the dissemination process of mathematics from its elaboration in sociocultural contexts until the moment when this knowledge, built and accumulated as a source of valuable cultural tradition, is reorganized in academic contexts (in scientific associations, etc.), under the aegis of science.

In the educational field, however, we realize that the curricular reforms and methodological reorientations currently in place in mathematics education seek to install a process of valuing mathematical knowledge experienced in the daily context. Even so, in the everyday practice of teachers,

> (...) the value and meaning of what is taught in schools are as far removed from everyday life as from scientific life. It is removed from everyday life because its achievement is not expected to serve for reflection and action in everyday life because to do that, people develop implicit models that serve to interpret the phenomena that happen in the intermediate dimensions of reality (mezzoworld), while academic knowledge tries to transmit—mainly, —scientific models and theories about micro and macro world dimensions (Arnay, 1998, p. 38).

Instead of talking about the superiority of one over the other, we should incorporate the idea of the coexistence of different ways of thinking generated to answer different purposes and needs. The conception of some scholars who defend the opposition between everyday and scientific aspects of knowledge is untenable. If we accepted this position, we could not explain the emergence of scientific theories throughout history. Thus, we must admit that scientific aspects can only be evidenced in knowledge if we take into consideration the individual's experience, that is, from the everyday aspect in which the knowledge was generated. Therefore, we can affirm that culture provides an essential contribution to the elaboration of scientific knowledge.

Likewise, Gómez-Granell (1998) asserts that the knowledge transmitted by schools is not everyday knowledge, but neither it is scientific knowledge and as for school learning, it also does not have the characteristics of discovery or scientific creation. Therefore, if we provide the organization of a challenging and inventive school environment, we will foster in students a creative spirit that will lead them to develop skills to seek their own discoveries and thus form habits and abilities that will be beneficial to them later in life, be it in the continuation of their studies or even in other socio-cultural activities. To accomplish that, we must understand that to move from everyday knowledge to scientific knowledge means to build more sophisticated, rational, and complex levels of both types of knowledge and to use them conveniently in the context in which they are needed.

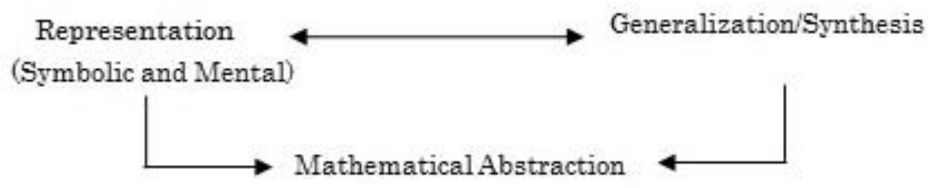
Arnay (1998) presents a favorable position and therefore assures us that the role of scientific knowledge in schools is not well established, because up until the current historical moment, there is no specific scientific school knowledge in the educational field. There is indeed a series of options adopted at the school level whose objectives are to transmit scientific information to train future scientists, create science consumers or perhaps people who are critical of scientific and technological development, etc.

As for the incorporation of scientific characteristics in school-level mathematics, we believe that the everyday aspect, the school aspect, and the scientific aspect of this knowledge are strongly evident throughout the historical path of its construction. The history of mathematics shows us that the product generated throughout this constructive process is the result of a transdisciplinary action that permeates these aspects. Although these three ways of understanding mathematics represent the sociocultural dynamics in which this knowledge is established as a human activity, it is in symbolic language that it reaches its informative fullness, that is, the communication of the entire established cognitive process.

It is, therefore, essential that we implement mathematics teaching that integrates into its pedagogical process the daily aspect, the school aspect, and the scientific aspect of the knowledge that is to be taught/learned in the classroom. This integration will take place through didactic activities that enable students to achieve the objectives proposed by their teachers. However, it is our duty to ask: how can this integration, this alliance, occur in the student's mathematical reasoning process? Is it, then, a productive activity in mathematics? How is this productive activity carried out?

It is common knowledge that mathematics is one of the school system's requirements to officially admit that students have achieved real and meaningful learning, whatever their school level. To briefly discuss this aspect of mathematical learning, we refer to the conceptions of Tommy Dreyfus (1991) when he discussed advanced mathematical thinking. Dreyfus (1991) states that there is a distinction between advanced and elementary mathematical thinking, marked by the level of complexity of each one and the way of dealing with it. He justifies that this distinction is evident in the modes of abstraction and representation of rational thought in mathematics. He also states that mathematical concepts are constructed by symbolic and mental representations, which, in the process of generalization or synthesis, make it possible to reach different levels of mathematical abstraction (Figure 5).

Figure 5



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Symbolic representations are essential for the organization of school and scientific mathematical knowledge, for they use a variety of codes available to individuals that help them to represent and organize formulations that express conclusive mathematical propositions about everyday experiences. This type of representation of mathematical experiences and reflections is manifested orally or in writing to communicate an idea, a notion, or a constructed mathematical concept. This codification, operationalized by symbolic representation, becomes an ideographic kind of writing that will constitute the field of the mathematical language of each individual or socio-cultural group. It is an essential tool in the definition of symbols of advanced mathematical thinking as proposed by Dreyfus (1991). However, the approaches employed in school mathematics often do not consider a connection that involves experience, interpretation, and mathematical coding of everyday activities. Thus, the symbolic representation does not express any meaning to the students, hindering the dialogue between a teacher and their classroom.

To weave our considerations regarding this type of representation, we present the example of an approach to the Pythagorean Theorem with students aged 14 to 16 years. This is an experience carried out in a public school, which showed us the different levels of symbolic representation on this subject. We identified a group of students limited to the ($a^2 = b^2 + c^2$) as algebraic expression of the theorem; another group that represented it geometrically, through the equivalence of square areas—both representations trivially expressed in mathematics books. A third group only associated the theorem with the right triangle through other forms of symbolic representation. At the end of the experience, it became evident that each symbolic representation seems to be linked to the number of schemas that each student had accumulated in their cognitive structure in order to express their perception of the subject.

Mental representations are expressed in the individual way that each person formulates their process of comprehensive internalization about any mathematical situation, be it everyday, school-level, or scientific mathematics. Such representations are established in the combination of each person's internal schemas and are usually the result of mathematical experiences lived by members of certain groups individually as well as among themselves. A single mathematical concept may have different mental representations due to the internal schemes that each person uses to interact with the outside world.

Based on the reflections announced by Dreyfus (1991), we can assert that a single mathematical concept's symbolic and mental representations are essential cognitive actions that mobilize internal combinations towards generalization or comprehensive synthesis to reach different levels of mathematical abstraction. Generalization requires a cognitive action of derivation or induction starting from specificities such as identifying common characteristics or expanding the domains of validity of particular conclusions. Synthesis corresponds to the combination or composition of parts as a way of composing a whole, which is often more than the sum of its parts.

This procedural movement of mathematical representation and abstraction constitutes a dynamic action that we qualify as constructive mathematical activity. As emphasized by Efraim Fischbein (1987), this type of activity involves three components: an intuitive, an algorithmic, and a formal one. The intuitive component concerns the way we use imagination, visualization, all our human experiences, and even our biological characteristics in the elaboration of mathematical thinking. Through intuition, we interpret situations that lead to the formulation of mathematical concepts. The algorithmic component refers directly to the use of algorithms in different schematic representations of a single thought. The formal component is linked to the use of ideographic writing that constitutes formal mathematical language and makes mathematical ideas accessible only to individuals who master such language. This component is considered a superior form of mathematical expression and therefore is used as an advanced way of presenting school mathematics.

The reflections presented by Dreyfus (1991) and Fischbein (1987) delineated explanatory axes of the construction of mathematical thinking from a socio-cultural to a scientific perspective, without, however, disregarding the school aspect of this elaboration. Based on our interpretation, we can establish a dialogue between the authors' propositions to connect them to the everyday, school-level, and scientific aspects of mathematics, as we've already discussed. To carry out such a dialogue the mathematical activities mentioned by

Fischbein (1987) must be introduced in the classroom as a teaching and learning proposal that integrates these three aspects of mathematics.

In the context of the historical development of mathematics, Fischbein's (1987) considerations provide us with enriching subsidies to interpret the characteristics of the transformations of the intuitive, algorithmic, and formal components of this cognitive and socio-cultural activity that gave rise to mathematical creation and its modes of representation and ideographic writing in the shape of formal mathematical language. This is because it is through historical information regarding mathematical concepts and relationships that we can understand the developmental trajectory of conceptual networks concerning mathematics. From this starting point, perhaps we will be able to understand what made it possible for society to grasp mathematical ideas built in different historical moments and how such ideas were organized and systematized in the form of theories.

CLOSING REFLECTIONS

Throughout this article, we have discussed and pointed out reflections concerning the relationships and connections between different conceptions regarding the production of mathematical knowledge in the interconnection of society, culture, and cognition. Therefore, as we come to a close, we can assert that the production of mathematical knowledge throughout our social history has been characterized by the constant creation and organization of codes to interpret and represent everyday situations in society. Over the centuries, it was transformed into formalized knowledge through ideographic writing that was expanded and disseminated in communication systems. It expanded into other institutional dimensions, as well as in representations as a formal language inserted in and validated by school systems.

From this reflection on the historical epistemological processes related to mathematics that involve interconnecting relations between society, culture, and cognition in the constitution of mathematics education, we infer that this knowledge was later incorporated into the cultural framework that we organize, institutionalize, and disseminate in society. However, the search for the historical reconstruction of mathematical knowledge starts to have significant pedagogical implications for the construction of everyday, school, and scientific knowledge of our students, if we use such historical information in a fresh perspective of production or conscious appropriation of the mathematical knowledge produced.

In this sense, the discussion on the construction of mathematical knowledge presented in this article leads us to admit that it is a process that is continuously in motion and is fostered by nature, society, culture, and by the interaction of individuals to understand and explain the world. This dynamic explains the transformation of society from its natural, social, and cultural context, thus providing the strategies of thought implied in the production of knowledge.

Starting from problematizing actions that involve internal reflections and representations of socio-cultural cognition, it is possible to establish research processes that function as ways of dealing with reality in and out of school, with the intention of relying on different ways of institutionalizing the constitution of mathematics education. This is, therefore, a way of expressing a problematizing dynamic materialized in investigative practices, based on the historical development of mathematics and its implications for mathematics teaching.

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