Japanese Lesson Study, its Nature and its Impact on the Teaching and Learning Mathematics

Masami Isoda
isoda@criced.tsukuba.ac.jp
https://orcid.org/0000-0002-0017-3935
University of Tsukuba
Tsukuba, Japan
Yuriko Yamamoto Baldin
yuriko@ufscar.br
https://orcid.org/0000-0001-7473-5657
Universidade Federal de São Carlos
São Carlos-SP, Brasil

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Abstract

This article aims to present a text summarizing the main aspects and details of the Japanese Lesson Study, which has been systematically disseminated to countries outside Japan since 2006, and after the publication of a book in English in 2007. The purpose of offering a text, in Portuguese, is to facilitate students´ and teachers´ access to knowledge about Lesson Study-LS and its importance for the teaching and learning of mathematics. The article presents the origins and role of LS throughout the century in the teaching of mathematics in Japan, discusses its impact on curriculum development in Japan in the past and today, the importance of LS for teacher education, the structure of LS activities, the Problem-Solving as a central concept for developing the mathematical thinking in the activities of LS, and makes some considerations of the recent movement of diffusion of LS.
Keywords: Origin of Lesson Study. Mathematics Curriculum Development. Mathematical Thinking. Teacher Knowledge. Problem-Solving.

Estudio de Clases Japonés, su Naturaleza y su Impacto en la Enseñanza y el Aprendizaje de las Matemáticas

Resumen

Este artículo tiene como objetivo presentar un texto que resume los principales aspectos y detalles del Estudio de Clases Japonés, que se ha difundido sistemáticamente a países fuera de Japón desde 2006, y después de la publicación de un libro en inglés en 2007. El propósito de ofrecer un texto, en portugués, es facilitar a los estudiantes y profesores el acceso al conocimiento sobre Lesson Study-LS y su importancia para la enseñanza y el aprendizaje de las matemáticas. El artículo presenta los orígenes y el papel de LS a lo largo del siglo en la enseñanza de las matemáticas en Japón, discute su impacto en el desarrollo curricular en Japón en el pasado y en la actualidad, la importancia de LS para la formación del profesorado, la estructura de las actividades de LS, la Resolución de Problemas como un concepto central para

desarrollar el pensamiento matemático en las actividades de LS, y teje consideraciones del reciente movimiento de difusión de LS.
Palabras clave: Origen del Estudio de Clases. Desarrollo curricular en matemáticas. Pensamiento matemático. Conocimiento del profesor. Resolución de problemas.

Lesson Study Japonesa, sua Natureza e seu Impacto no Ensino e Aprendizagem da Matemática

Resumo

Este artigo tem como objetivo apresentar um texto resumindo os principais aspectos e detalhes da Lesson Study japonesa, que vem sendo sistematicamente divulgado para países fora do Japão desde 2006, e após a publicação de um livro em inglês em 2007. O propósito de oferecer um texto em português é facilitar aos alunos e professores o acesso ao conhecimento sobre a Lesson Study-LS e sua importância para o ensino e aprendizagem da matemática. O artigo apresenta as origens e o papel de LS ao longo do século no ensino de matemática no Japão, discute o impacto da mesma no desenvolvimento curricular no Japão no passado e hoje, a importância da LS para a formação de professores, a estrutura das atividades de LS, a Resolução de Problemas como um conceito central para desenvolver o pensamento matemático nas atividades de LS, e tece considerações do movimento recente de difusão da LS.
Palavras-chave: Origem da Lesson Study-Pesquisa de Aula. Desenvolvimento curricular em matemática. Pensamento matemático. Conhecimento docente. Resolução de problemas.

Introduction

The main objective of this article is to present the Lesson Study-LS to the Portuguese language community with a text accessible to educators and teachers of mathematics of basic education, presenting the main aspects and details of the Japanese Lesson Study, using as a basis the references that spread this methodology outside Japan. As these references were disseminated in English, initially, and in Spanish to the community in Latin America, not all the literature produced in the last two decades has been studied or worked on in Brazilian schools as implemented practices. The increasing diffusion in recent years of the methodology in Brazil and Portugal, as an important research topic of Mathematics Education, made the project of translating into Portuguese to contribute to deepening the basic knowledge of the Lesson Study very significant, which motivated this article, first in English as a basis to a Portuguese version.

In this introductory section, we briefly describe the movement, at the turn of the twenty-first century, to bring in a systematized way the knowledge of Lesson Study-LS to the United States, when in the last decades of the twentieth century, there was already collaboration between the mathematics education research of Japan and the United States, which aimed at improving the learning of mathematics in the classrooms of both countries, for example, in the 1960s, as stated in the preface of (Becker & Shimada, 2007).

The present article relies on parts of the basic literature, in English, which gave more intense openness to the diffusion of LS outside Japan, such as (Isoda *et al*., 2007), but it brings as well other references to update the literature review.

By the year 2000, the national document of the US, "Before It's Too Late: Report to the Nation from the National Commission on Mathematics and Science Teaching for the 21st Century (2000)," was released. This document quoted TIMSS Video Tape Study and mentioned, "The basic teaching style in American mathematics classrooms remains essentially what it was two generations ago. In Japan, by contrast, closely supervised, collaborative work among students is the norm". Since then, the Japanese Lesson Study (LS), Jugyou Kenkyuu, became the US movement in education and intensified its influence on the world.

In this process of influence, several misunderstandings about the essential concept of LS have emerged, preventing the effective implementation of the methodology. For instance, it is noted that several pieces of information would have been lost on the adaptations depending on the challenges researchers faced in various countries. On this issue, Watanabe (2018) points to studies such as (Chokshi & Fernandez, 2004), which highlight the misconceptions and conceptual misinterpretations of LS that emerged in the initiatives of implementation of LS in the United States in the first decades of the twenty-first century, when such initiatives focused on the procedural aspects of LS, caused by lack of in-depth information on the principles of the methodology. Care with the cultural perspective of the LS activity, as already pointed out by Stigler and Hiebert (1999), proved to be an important element to consider when planning the expansion of the methodology in cultures outside the country of origin.

In this scenario, the Asia Pacific Economy Countries-APEC-LS project was proposed by Thailand and Japan around 2006, promoting a collaborative movement among APEC participating countries to implement the Japanese LS. In each year, from 2006 to 2018, a theme was specified within the dimensions of the LS to promote efficient dissemination to support education and educators, especially from developing countries in the region, with an annual meeting promoted by the University of Tsukuba (http://criced.tsukuba.ac.jp/math/apec/).

The APEC-LS Project aimed to promote appropriate notions of LS and to support improvement movements in the teaching and learning of mathematics in the participating countries. In the publication of the thematic issue on LS, several initiatives and projects of this movement are described by other authors in this thematic publication.

In this article, we will explain Japanese LS initially from the historical perspective of its origin, its functions, and the teaching approaches for sharing appropriate notions of LS. We will discuss the potential of LS to impact the curricular reforms of mathematics teaching aimed at adapting them to the updated demands of education and society.

1. Origins, History, and Variety of Lesson Study

1.1. Adaptation of Western Culture in Japan

Under the seclusion policies and class system that characterized the Edo period for about 260 years prior to the installation of the new Meiji government in 1868, literacy (and numeracy) education was available to commoners through *terakoya*, temple schools, that had opened autonomously around the country. Commerce thrived, and the class system in the social organization gradually collapsed during this period of seclusion, and by the late Edo period, individual knowledge and skills were highly regarded in the recruitment of workers. Due to the widespread emergence of temple schools, to which parents could voluntarily send their children, the literacy rate at the end of the Edo period was 43% among males and 10% among females, even then making Japan one of the most educated countries in the world. Individualized instruction was the common teaching method employed. (Isoda *et al.,* 2007)

In 1872, the Meiji government issued the Education Code and, at the same time, established a teacher education school (Normal School) in Tokyo (forebear to the University of Tsukuba). With the objective of disseminating Western scholarship, the government invited foreign teachers of different subjects to teach Western ways of conducting classes to the schoolteachers. The foreign teachers introduced the concept of whole-classroom instruction, a style then still rare even in the West, into the Normal School, as can be seen in Figure 1. At that time, the Japanese teachers and students were familiar only with the individualized instruction model in which subjects were taught individually based on the academic abilities of the student. The teachers learned with new classroom dynamics not only the contents of the subject but also methods of teaching by observing their teachers' behavior.

Figure 1: Shift between the classroom scenarios before and after the Meiji Revolution

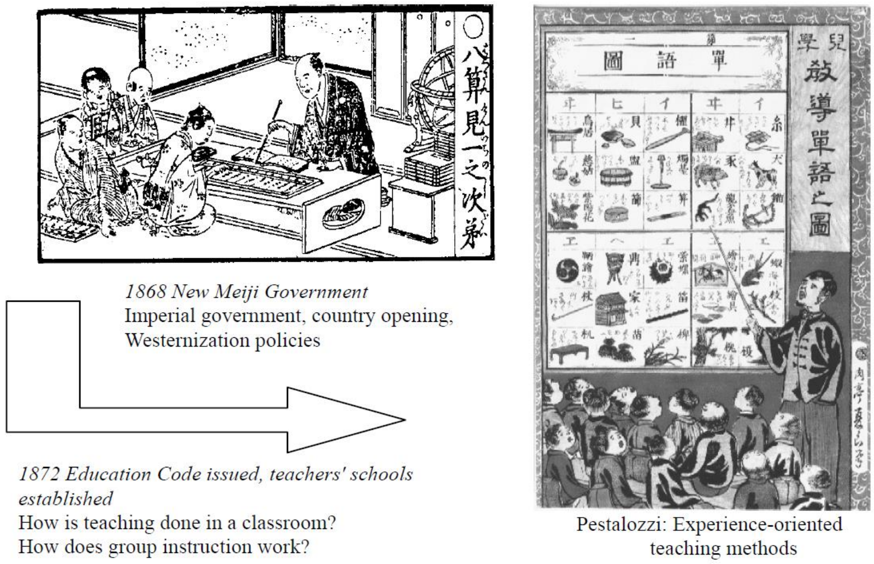


Figure 1: Shift from the curriculum and teaching methods of the *terakoya* (temple schools) to those of new types of schools.

Source: (Isoda *et al*., 2007, p. 9)

Textbooks created by foreign teachers at Normal School contained drawings of students raising their hands to answer questions posed by the teacher, as shown in Figure 2.

Figure 2: Comparison of illustrations in textbooks before and after Meiji

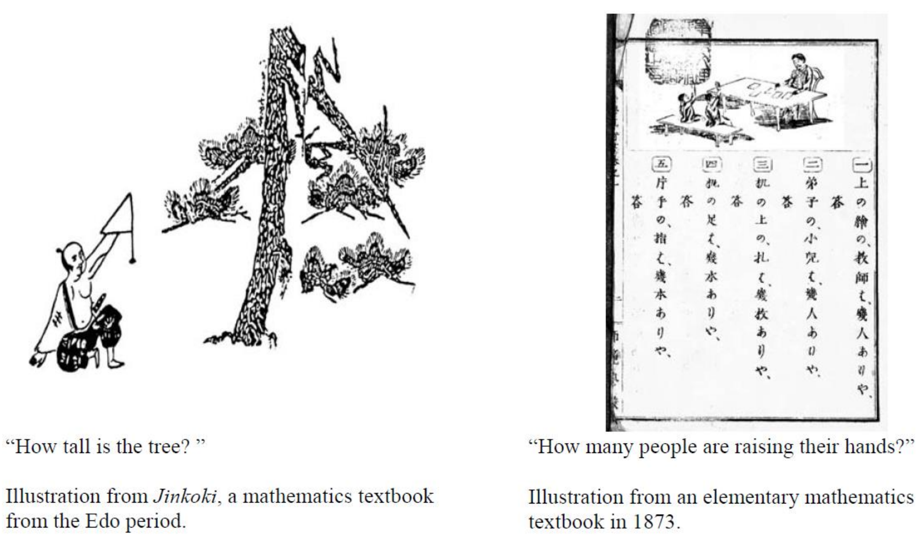


Figure 2: From textbooks (left) that allowed students to study numeracy at their own discretion, depending on their needs, to textbooks (right) designed to allow students/teachers to simultaneously study leaning/teaching methods.

Source: (Isoda *et al*., 2007, p. 11)

The illustration on the left of Figure 2 represents an exercise from the mathematics textbook, *Jinkoki*, from the Edo era, before the Meiji government, in which a text of a problem is posed for the student to solve it individually, using his own knowledge. On the right side, we can see a question posed "How many students are raising their hands?", it is also the proposition of a problem, but its statement contains a question to be worked with the figure and dialogue, representing different pedagogical approach to the knowledge of teachers at the time. The foreign teacher wrote a textbook that teaches instruction methods as well as mathematics at the same time.

The group instruction model implemented at the Normal School in Tokyo spread to other teacher education schools around the country. Due to financial difficulties under the new government all the teachers' schools eventually closed around 1880, except the Normal School in Tokyo which became Higher Normal School in 1986. During the decade while the schools were open, the practice of group instruction was disseminated around the country by graduates of the teachers' schools. For the instruction in classrooms, they used scroll pictures as on the Figure 1, right, and textbooks with illustrations as on the Figure 2, right.

As Smith and Mikami (1914) wrote, Japanese mathematicians had already achieved highest level of the mathematics knowledge by 17th century. At that time, some of their findings were even earlier than those of western mathematicians through the renovation of Chinese mathematics written with Japanese and Chinese notations and tools such as abacus.

At Edo era, Japanese textbook for children already included numerous pleasant pictures and cartoon stories for enjoyable teaching and learning like in Figure 2, on left. The elementary mathematics textbook, *Jinkoki*, was the best seller among the publications through Edo era and to avoid bootlegs, it began to include challenge problems in the editions. This initiative supported the establishing the custom of posing mathematical problem each other. This custom spread to common people to encourage mathematics learning and research. On these bases, Japanese educators made it possible to introduce and adapt Western mathematics and education to new social organization.

1.2. Initiation of Lesson Study in Japan

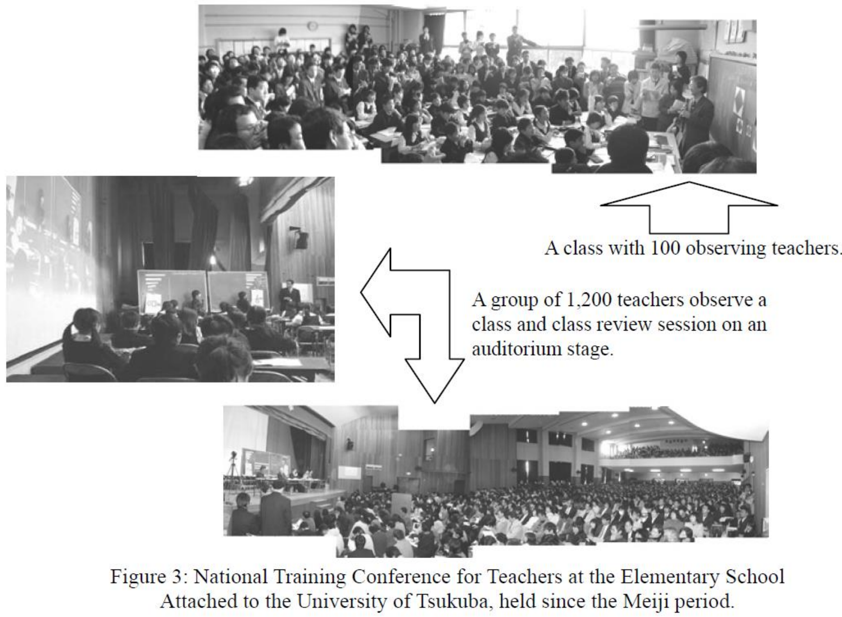
Kyoushi Kokoroe, was the document that established the regulations of teacher's knowledge and profession, published by the Normal School (1873). It described, for instance, that other teachers may come into a class under the class teacher allowance, being such rule already a tradition from the beginning.

In the years 1880, study about the group instruction and its dissemination reached new heights when the overseas study missions began returning to Japan. At the same time the foreign teachers were working in Japan to teach new forms of instruction and content subjects, study missions composed by Japanese teachers and educators had been sent abroad to bring lively knowledge and teaching methods in their return.

Then, mission delegates became teachers at the elementary school attached to the normal school after their return, and a book for innovation of the teaching method was published (Wakabayashi & Shirai, 1883). This book contained the orientations to prepare teaching materials, as well as the instructions for conducting classes/lessons, and the ways of observation and critique. From the beginning of LS, it already included the current format of LS. Additionally, it included the model protocol amongst class-teacher and students as for the manner to re-present the lesson. The dialog is currently known as dialectic however their question-and-answer style was like traditional *Zen* or Confucius dialog. The Japanese Ministry of Education also enhanced Pestalozzi method by using hanging scrolls for showing pictures to illustrate the contents and problem-situation. Under the instruction of the Ministry of Education, the methods which include how to develop lesson plan, implement lesson and critique after class were implemented throughout Japan as a model.

Open classes, the origin of research-lessons, were held to encourage the proposal of new teaching methods and teaching curricula, producing the first interactive LS groups initiated through the government and the elementary school, attached to Normal School.

Figure 3: Scenes of open classes for LS, a tradition in Japanese mathematics education



Source: (Isoda *et al.*, 2007, p. 13)

Figure 3 shows one of the national teachers' training conferences by the elementary school, which have been held since the Meiji period. In the case of the elementary school, LS at Auditorium in Figure 3 is only done for Mathematics and Japanese language. Especially, mathematics is the major subject for LS in Japan because the school level LS in teaching mathematics usually demonstrates well how teachers develop children´s growing up in their learning abilities.

1.3. Historical Development of Lesson Study

As the country grew wealthier, it became possible for anyone to graduate from elementary school. The normal school became the Higher Normal School in Tokyo which produced the teacher educators, and every region re-established their normal schools. The custom of LS was shared through the internship program at normal schools.

At the beginning, Pestalozzi's method was the sole subject worked for LS. However, in later years teachers and teacher educators began to catch up newest western reform issues and Japanese innovative ideas for new themes of LS, which were related with curricula reforms, as shown in Figure 4.

Figure 4: Table of theme-topics of Lesson Study

	Topic of Lesson Study	Remark
1880s	Pestalozzi Method and Dialog Method (including argumentation/discussion/dialogue between teacher and students)	Not only limited to mathematics
1910s	Mathematics for Life (including problem posing)	Not only limited to mathematics
1930s	Curriculum Integration in Mathematics (including Open-Ended Problems)	From the 1900s
1950s	Core curriculum movement based on social studies	Under the occupation after WWII
1960s	Mathematical Thinking (Japanese way of New Math)	Related with New Math
1970s	Open-Ended Approach and Problem Solving Approach	For developing Mathematical Thinking
1980s	Problem Solving	Related with the U.S.

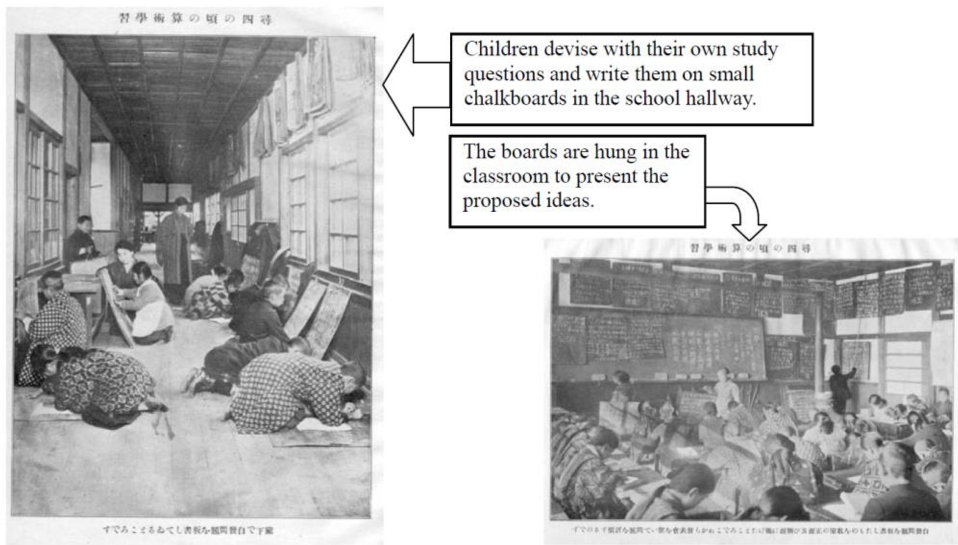
Source: (Isoda, 2015)

In 1904, Journal of Education, the oldest Japanese Journal in Education, was published by the elementary school. In whole Japan, teachers have been able to catch up the newest reform issues by themselves through the Journal, in which mathematics was included.

In addition to the format provided by Wakabayashi and Shirai (1883), texts for LS usually clearly indicate the study theme of the lesson. Currently, if the study theme is not written on the lesson plan, it does not function as “lesson plan of LS”, even that the elaboration of a lesson plan is necessarily studied in ordinary professional development activities.

After Wakabayashi and Shirai (1883), next large movement in Japan was around 1910s, in the case of elementary school, with new teaching methods based on the educational philosophy of scholars such as John Dewey. This movement launched an era in which private school teachers began proposing their own teaching methods. At that time, a new teaching approach to develop children who learn by and for themselves was proposed. It allowed students to come up with their own study questions, discuss with one another whose question they wanted to research, and then go about researching the selected question. This problem posing approach is illustrated in Figure 5.

Figure 5: Study on how to teach students to develop their own study questions, at the Elementary School of Nara Women's Higher Normal School, around 1920.



Source: (Isoda *et al*, 2007, p. 15)

Beyond catching up the Western customs, around 1920s were the moment to establish such original and innovative teaching approaches that focused on problem-solving, which today are globally recognized as models of constructivist approach.

In secondary level mathematics, the Klein Movement from the beginning of 20th century which advocated the integration of independent subjects of mathematics into one mathematics curriculum was also well known in early stage in Japan. However, Japanese mathematician group of the University of Tokyo wished to keep traditional subjects in mathematics and Higher Normal School's math-educators became independent through the establishment of the Japan Society of Mathematical Education (JSME) in 1919 (Isoda, 2019).

From around 1910s up to World War II-WWII, the period that comprises the difficulties of Great Earthquake in Tokyo (1923) and Global Depression (1929), Japanese Curriculum Standards were revised based on the LS by Higher Normal School's teachers and mathematicians, with collaborations of the members of JSME. Until WWII, several journals for mathematics education which promote LS and curriculum development had been published, to be discussed in the next section.

After WWII, Japan was controlled under the General Headquarters of US that occupied the country, for 10 years. The Education Standards became just a recommendation from the government, and the Normal School became the teacher education college, which main teacher trainers were subject specialists such as mathematicians. At the same time, the established custom to develop mathematics curriculum through LS was supported by mathematicians and math-educators, providing the bases for Japanese New Math Movement through their contributions. At the primary school level, the ideological oppositions to it beyond the mathematics remained until 1980s because there were few math-major teachers in primary schools, many of them concentrated in how to teach mathematics instead of how-to develop children who learn mathematics by and for themselves, as recommended as main issues on the standards.

It is interesting to notice that even though the discussion had gone parallel between sides, teachers` core value was making efforts for children. In this regard, LS clearly demonstrated the difference, showing that adequate approaches in classroom were possible through lessons to develop children actively and enjoyably with planned activities. The Problem-solving approach (teaching through problem solving) has been strengthened since that times because it clearly demonstrated children growing to learn mathematics by and for themselves, which evidence was investigated by the school through LS.

In 1980s, the official system for training first year (novice) teachers was established in the recently launched *in-service teacher education curriculum,* in which the Problem-solving approach was offered as attractive and meaningful methodology for young teachers. Since then, problem-solving approach is well known as a major way of teaching mathematics in Japan.

2. How Lesson Study spread to the world

The first international survey by the International Association for the Evaluation of Educational Achievement-IEA was done in 1964, and Japanese students got highest average. Then, in 1980s, researchers from many advanced countries visited Japan for knowing the reason and the how. In 80s, Ministry of Education has begun to offer the teacher education program for foreigners, for 18 months, attending around 100 teachers every year. At that time, Japan International Cooperation Agency-JICA also began to dispatch Japanese teachers abroad and invited foreign teachers for training to collaborate in dissemination of good education. First Japanese teacher who went dispatched to foreign country with the support of JICA was an elementary schoolteacher. He went to the Southeast Asia Ministers of Education Organization-SEAMEO, Regional Centre for Education in Science and Mathematics - RECSAM in Malaysia which is the teacher training center for Southeast Asia in Mathematics and Science. One part of the building and the equipment of RECSAM itself was also the Japanese Official Development Assistance.

In mathematics education research, Japan-US comparative study for problem solving has been done from the middle of 1980s to the middle of 1990s, and several US researchers had the opportunity to observe Japanese mathematics lessons. This has enhanced the previous collaboration between Japanese and US researchers in mathematics education as already mentioned in the introduction of this paper. On this occasion, *Jugyou Kenkyu* was translated as Lesson Study-LS, when the TIMSS video tape study compared the lessons of US, Germany and Japan, and Makoto Yoshida wrote, in 1999, his famous PhD thesis on this context, being (Fernandez & Yoshida, 2004) a book elaborated upon his work.

In Philippines, since 1994 JICA has provided 87 technical assistance projects on mathematics education for developing countries which have been changing in time, depending on their developing status. On this background, "Before It's Too Late: Report to the Nation from the National Commission on Mathematics and Science Teaching for the 21st Century" (2000) was published in US. As mentioned in the introduction, this document became the trigger of the dissemination of LS towards the world. Also mentioned before, the APEC lesson study project was established since 2006 with participant economies of APEC and has promoted LS under specific themes of LS for each year (http://criced.tsukuba.ac.jp/math/apec/).

3. Variety of Lesson Study

In the world, under the influence of Yoshida and others, LS became known by the cycle of "planning (lesson), doing and seeing" process, with collaborations of various agents and participants. The start of LS in each country is in general done by researchers.

The known Japanese model for LS is usually focused on school-based Activities to establish a learning community. In Japan, LS has been a part of teachers' profession since the origin which process was already characterized by Wakabayashi and Shirai (1883) until 1980, when the official *teachers' professional development regulation reform* set LS as the part of the curricular obligation for first year teachers and for tenth year experienced teachers at in-service teacher training centers. Before this reform, study and training were just the right for teachers, and LS was a voluntary part even when it was supported officially. They do LS by and for themselves with various background supporting system.

About the background of LS, we can say that there is various support system.

Firstly, there is an official support system, composed by school principal, supervisors, and teacher trainers, which promote the establishment of a learning community in the school. Selected teachers are also supported to go to master program in education at universities. In these programs, LS is usually a part of their education.

Secondly there is unofficial support system such as teachers' community. For example, there are various teachers' journals, guidebooks and private articles published by teachers' groups, society, and publishers. Most of them are written by teachers as part of their cultural practice. When we compare these writings with academic journals in educational research with qualified reviewing system, a major difference is that there is no custom or necessity to write the articles in accordance with academic styles such as indication of references, etc. It is written to share the good or necessary ideas of teaching and the objective of writing itself is to enable to others reproduce better teaching for themselves. Thus, the descriptions they write are often unnecessary to follow exactly their lesson itself. But it should be written based on their reflections what they learned from children in the lesson during some cases of open class (public class), and from other teachers in the post-lesson discussions. The opportunity for contribution to journals and guidebooks are usually given by editors or others. It is an honorable experience for young teachers because it means that they are recognized in teachers' community as outstanding practitioners for developing children. Asking others to contribute to journals has been functioning to enlarge LS community for well experienced teachers. Even researchers support their publications as the editors, though they do not always attend every LS in their LS community.

Depending on school levels, LS are not the same. Internationally shared LS is known through Japanese problem-solving approach for teaching and learning mathematics at the primary and the middle (Junior high) school level (Isoda & Katagiri, 2012), because their practices are possible to share. On the other hand, for the case of high school level, although there is a regulation to do LS on the official professional development system, open classes (public classes) for LS can be seen only in special occasions. High school mathematics teachers have been focused more on their own curriculum development because the students go to different schools depending on their achievements and the professional orientation of the curriculum for the chosen careers. Therefore, the practices of different lessons are not easy to share at this level because teaching content could not be the same in different schools nor have the same learning objective. However, the teachers do practice the collaborative working through team-teaching, following the "planning, doing and seeing" process with collaborations. They also can differentiate students depending on their achievements, even when they have to teach more classes than their obligation.

4. Curriculum Development through Lesson Study in the Past and Nowadays

In this section we show some cases of curriculum development in historical order across generations. It is important to point that LS clearly promote the renovation of the textbooks and supplementary books for students and teacher educations as well as mathematics curriculum.

4.1. First Textbook as the result of Lesson Study

In the case of primary school mathematics textbooks, one national textbook which focused on to acquire arithmetic operations had been used from 1905 to 1934. Although it was revised slightly in three occasions, several proposals based on the achievements from the practice of LS were not reflected in textbooks. In this context, LS was only functioned to produce innovative methods of teaching. To promote the development of mathematical science thinking, first major revision of the textbook was published in 1935 starting from the first grade and completed in 1940 with the contributions of Elementary School Mathematics Teachers Group.

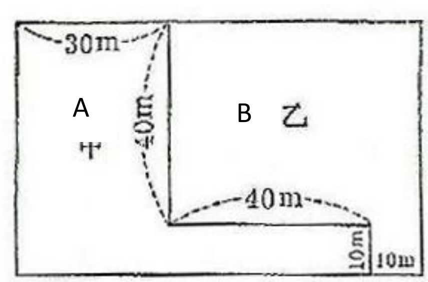
Figure 6 is an example of a problem in this publication. In current view, it is an Open-ended problem which has not enough condition and possible to produce various answers. It also looks like OECD PISA type problem.

Figure 6: A sample of problem from primary mathematics textbook

Problem

A and B have land adjacent to each other as shown in the diagram on the right.

Both of them would like to make each land simple rectangular without changing the area. How shall they fix the boundary.



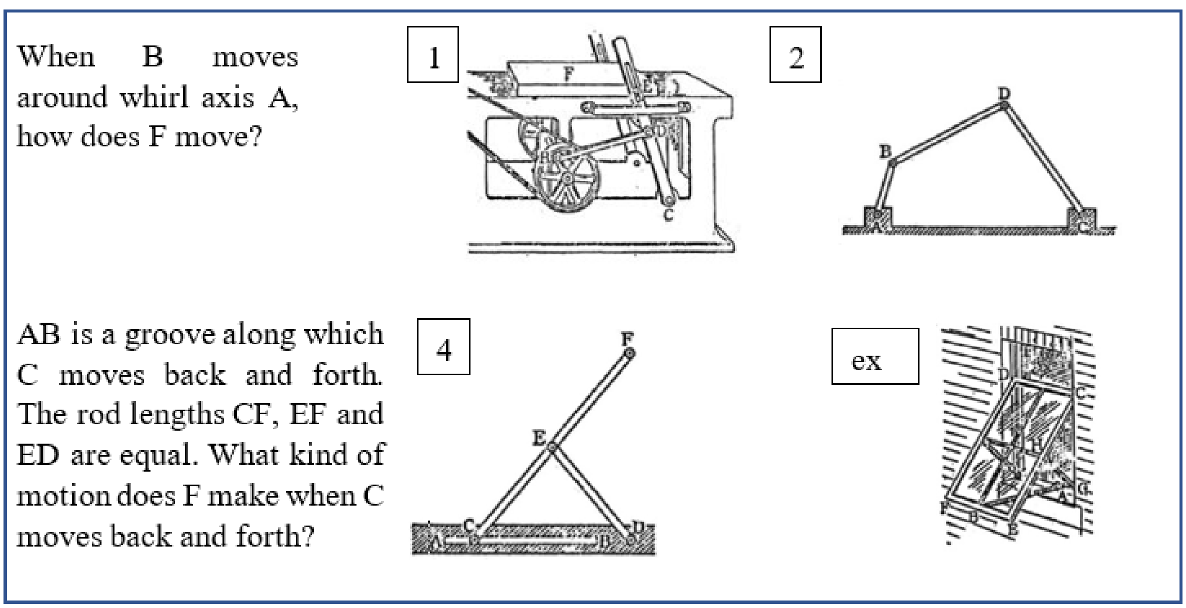
Source: http://www6.plala.or.jp/maeda-masahide/4menseki.html

After the revision of primary textbook, the corresponding middle school curriculum reform was proposed on the annual meeting of JSME in 1935. At the same time, Ministry of Education established the curriculum reform committee. Under JSME, the proposals were discussed in Tokyo. Osaka and Hiroshima committees independently based on each LS worked under the discussions and research in their related journals. In the case of Tokyo, mathematics teachers' group of Middle School in Higher Normal School at Tokyo belonged as members of both committees, and they finally engaged major role for the revision of national curriculum (1942) and textbooks (1943). The textbooks were first mathematics textbooks which integrated various subjects in mathematics as one unified mathematics concept, under the influence of the Klein movements. The principle of the textbooks was to promote mathematization by students and it was written based on the practice of LS at Middle School.

For example, there were task sequence to learn Mechanism like in Figure 7 which provided geometric bases for Calculus in Figure 8.

Later viewed as such by Freudenthal, H. (1973), the principle of mathematization means reorganizing mathematics by using functional representations.

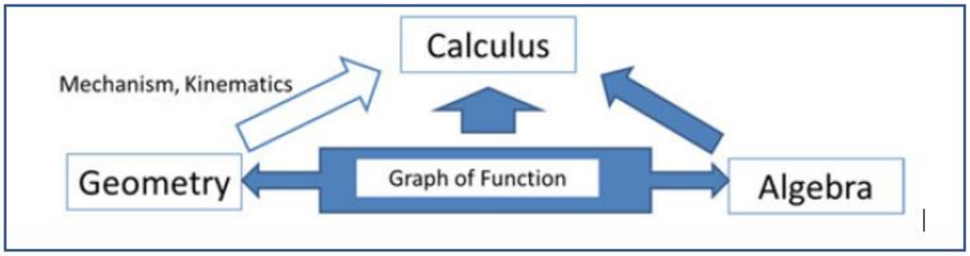
Figure 7: Analysis of the Motion on Mechanisms



Source: Adapted from Bussi, Taimina & Isoda (2010)

(Isoda, M., 2018). https://link.springer.com/chapter/10.1007/978-3-319-99386-7_9

Figure 8: Integration of Geometry and Algebra with Functions to Calculus



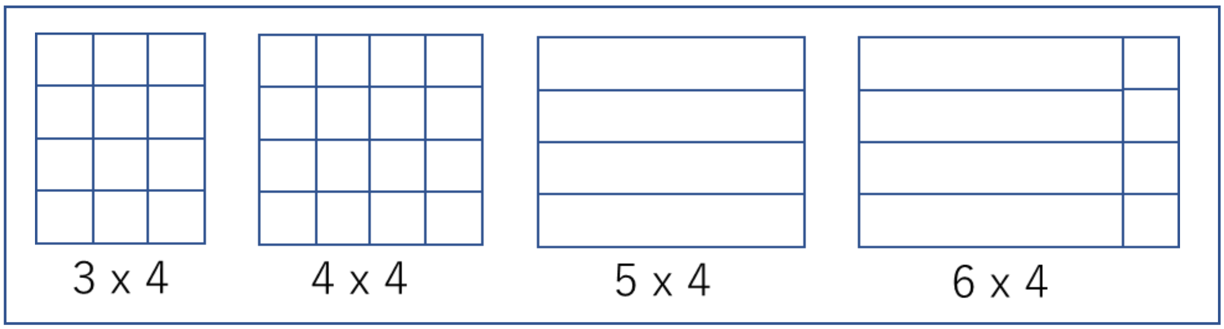
Source: (Isoda, 2019, p. 126)
(Isoda, M., 2018). https://link.springer.com/chapter/10.1007/978-3-319-99386-7_9

The examples resulted from the LS at Middle School and there was a remarkable episode: After WWII, a teacher named Tanaka who was the author of the textbook returned to the countryside, Gifu, to take over his father´s temple and became a teacher at Gifu High School. The impression of another teacher who observed his LS was "He just walked around the desk for supporting students and didn't teach." Observer teachers who attended LS were expected to learn the way of lecturing, however his methodology, which is currently called problem solving approach, was too outstanding for them at that time, because his approach was oriented to develop students to learn mathematics by and for themselves by using prepared task-sequence.

4.2. Principle for the Curriculum: Extension and Integration, or General to Special

After the reestablishment, the national curriculum standards had been criticized by teachers' unions for 30 years. Meantime, with critics and against critics, LS produced theories for curriculum and teaching. For example, the government set the extension and integration principle which explains a task sequence that went against the general-to-specific principle proposed by the LS group with a mathematician Hiraku Toyama. This principle was used by the group since the 1950s with the name of "the water supply method", as a metaphor from general-to-specific (Kobayashi, 1989). General to Specific was the system of mathematics for the group and it was the theme of their LS. For teaching general idea, they proposed a specific use of manipulatives in general, and theorized their teaching as "the internalization of Schema". This word was borrowed from Piaget to explain the mathematical abstraction of introducing new operation based on existed operation, but their theory was hiddenly and strongly supported by materialism because they denied the contradiction which enhanced on Piaget's genetic epistemology. They never discussed the cognition through 'accommodation' beyond contradictions on their LS. On their own theoretical bases, they proposed to explain "the general" by using tiles: single, five, ten (two-five), square (ten two-fives), 10 square, 100 square … (they did not go to 3D cube). On their rule to use manipulative tiles, they must change the five singles  to the five-tile  because the tiles function as intermediate-tools between fingers and numbers. They believed that the rule should be applied even in the case of multiplication, as illustrated in Figure 9.

Figure 9: At 5x4, the singles should be changed to the fives. Then, 6x4 looks 4x6.



Source: (Isoda, 2019).

The Figure 9 indicates strange things like the counting unit changing from four to six. The picture on the right of Figure 9 is shown as being still 6x4, and on their teachers' guidebooks, it says that it is the moment for teachers to explain to children that "Multiplication is not addition." They asserted that their method is correct because it is mathematically well configurated within primary level. As far as one believes that the tile management rules as in the Figure 9 are correct, it can be recognized that we are only considering the area of tiles in total as the answer for multiplication briefly, and one never sees how each line of tiles would express the model to answer the result of each row multiplication. Against such critique, the belief of that group was strong enough to some of them continued their LS with this approach for their classroom children, until their retirements, for more than 30 years in some limited regions such as Hokkaido, north island of Japan, which was in front of Soviet Union.

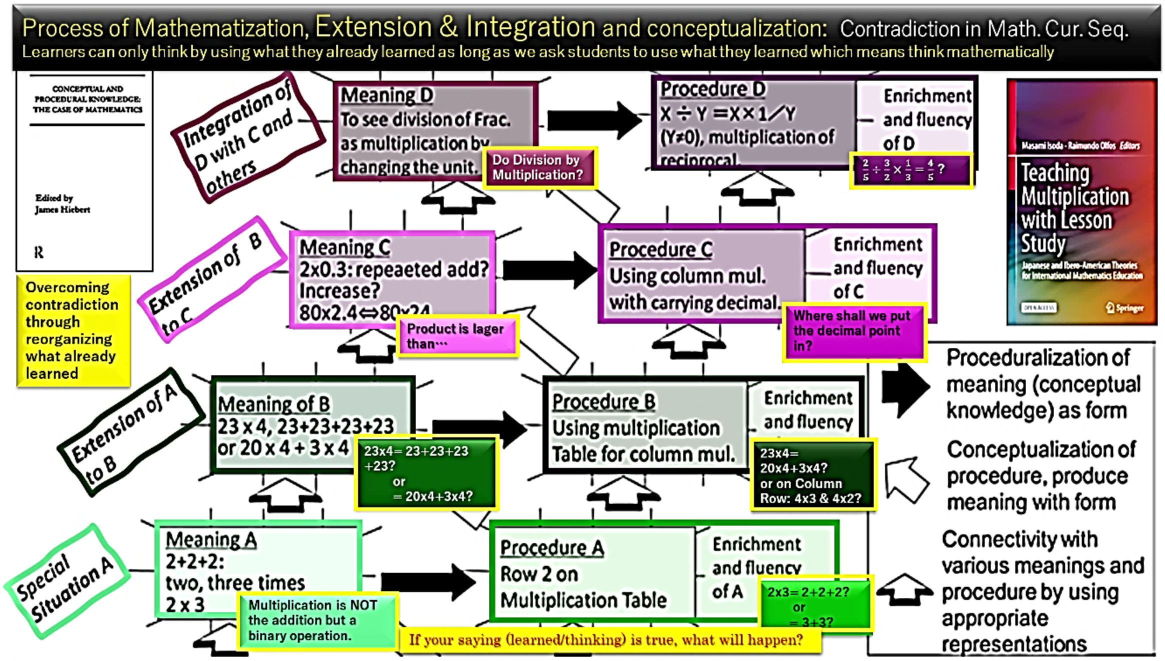
They denied the national curriculum standards in course by that time because it produced some misconception through over generalization by children. Misconception was just a bad thing for them. As counterpart, several LS groups from the government side had to make clear the principle and the theory against the "water supply method". Some groups called it mathematization and other group discovery. Ito (1968) named his theory as discovery methods, which were proposed to use specific representations to mediate ideas such as proportional number lines. This idea is currently well known as Japanese approach and is proposed to teaching mathematics, for it enables children to use these representations by and for themselves (Ito, 1971, English version). This was the result of his LS group.

Under the confrontation of these counter theories, National Standards set the principle 'Extension and Integration' since 1968, which implicated the overcoming of misconception beyond the over generalization, and it was considered as basic for teaching with mathematical thinking. With this principle, the misconception is recognized as good thing within the expectations in national curriculum, because it can be the result of overgeneralization resulted from the children´s thinking by and for themselves using what students already learned.

Figure 10 was later proposed by Isoda (1992) and Isoda and Olfos (2021) to explain the process of extension and integration in the case of multiplication, by using the theory of conceptual and procedural knowledge (Hiebert, 1986). Here, Isoda applied the theory to explain the curriculum and textbook sequence. Conceptual knowledge is usually taught for bringing meanings. However, it needs to use some known form of procedure. After introducing the meaning of multiplication as a binary operation (expression), the multiplication table is proceduralized from repeated addition; otherwise, students cannot distinguish it from addition as a new operation. In the process of extension and integration, inconsistencies usually appear.

For example, for multidigit multiplication, students need to see the multidigit numbers under the base ten system to apply the multiplication table instead of just repeated addition. For the extension of multiplication to multidigit numbers with column methods, multiplication as repeated addition should be integrated with the base ten system by using the rule of distribution. If we extend multiplication from whole numbers to decimals, the product of multiplication becomes small in case. It cannot be explained well as repeated addition. In the Japanese textbooks and Japanese teachers' lesson design, as shown in (Isoda & Olfos, 2021), these processes are discussed more precisely in the classroom practices in relation to the task sequence on the textbook.

Figure 10: Through extension of number, meaning and procedure are re-integrated.



Source: (Isoda, 1992, 1996, 2009)

The parallel discussions between Toyama Group for Union side and other Groups closer to government side well demonstrated their different views of Mathematics. Toyama view related with more Platonism as for ontology of Idea, and the discussion for internalizations from physical world is more oriented to Aristoteles. Government side was more oriented to the dialectic epistemology by Plato and Hegelians for overcoming contradictions.

In Japan, New Math movements began in 1950s and enhanced in 1960s. Through the period, mathematicians used to strongly support their LS groups. In the case of Hokkaido, mathematician Akitsugu Kawaguchi who was the president of the Hokkaido Society of mathematics education, supported his study groups in relation to the implementation of the annual meeting of JSME at Sapporo (1961) in Hokkaido. The theme of their LS was innovation of geometry curriculum from the perspective of Dynamic Geometry, meaning transformative geometry. His LS group produced their proposal depending on their school levels. Primary school group published the guidebooks for teachers on how to teach the idea of transformation at primary school from the editorial Meijitosho, a teachers' guidebook-publisher. Middle school group published four experimental textbooks, under the textbook company, Kyouiku Shuppan, which explained the mathematical transformation. It was the result of their learned Dynamic Geometry which oriented curriculum development, but quite difficult to implement it at their school level. High school group published the guidebook for students from the exam guidebook publisher, KeisetuJidai-Obunshya, about how to apply the ideas of Dynamic Geometry to entrance-exam problems to university. Those three different products also demonstrate the difference of LS at each school level, at the era of New Math in Japan.

5. Mathematical Thinking and Values for the Objectives of Lesson Study

Japanese aims of education have been described as three pillars: human character formation (such as values and attitudes), general thinking skills (such as mathematical thinking and ideas), and specific knowledge and skills (such as mathematical knowledge and skills). Even if we change the terminology, the principal aims are found as common not only for Japan but also for other countries such as the Southeast Asian countries (Mangao *et al*., 2017). These aims were generally discussed until New Math era.

The first two pillars are usually explained as higher order thinking skills in many countries and as the learning content for *learning how to learn*. It is usual for teachers to write or share these objectives through the lesson plan. According to the Japanese principle of the national curriculum, these aims are symbolized by a single concept: "*Developing students who learn mathematics by and for themselves*" (Shimizu, 1984).

In Japanese mathematics education, this has been recognized in relation to mathematical activities as for reorganization of living and life (Ministry of Education, 1947). As a concept, the activity has been re-explained as mathematical thinking and attitude (Ministry of Education, 1956) by Japanese math educators, who have tried to explain it further.

Figure 11: Katagiri's Framework for Mathematical Thinking

I. Mathematical attitudes: Mindset
1. Attempting to grasp one's own problems, objectives, or entities clearly by oneself
(a) Attempting to have questions
(b) Attempting to be aware problematic
(c) Attempting to find further problems from situation
2. Attempting to take logical-reasonable actions (reasonableness)
(a) Attempting to take actions that match the objectives
(b) Attempting to establish a perspective
(c) Attempting to think based on the data that can be used, previously learned items, and assumptions
3. Attempting to represent matters clearly and simply: Clarity
(a) Attempting to record and communicate problems and results clearly and simply
(b) Attempting to sort and organize objects when representing them
4. Attempting to seek better ways and ideas
(a) Attempting to raise thinking from the objects to operations
(b) Attempting to evaluate thinking both objectively and subjectively, and to refine thinking
(c) Attempting to economize thought and effort
II. Mathematical thinking related to mathematical methods: Mathematical Ways of Thinking
1. Inductive thinking
2. Analogical thinking
3. Deductive thinking
4. Integrative thinking (including extension)
5. Developmental thinking
6. Abstract thinking (thinking that abstracts, concretizes, and idealizes, and thinking that clarifies conditions)
7. Thinking that simplifies
8. Thinking that generalizes
9. Thinking that specializes
10. Thinking that symbolizes
11. Thinking that represents by numbers, quantities, figures and diagrams
III. Mathematical thinking related to mathematical contents: Mathematical Ideas
1. Clarifying sets of objects for consideration and objects excluded from sets, and clarifying conditions for inclusion (the idea of sets)
2. Focusing on constituent elements (units) and their sizes and relationships (the idea of units)
3. Attempting to think based on the fundamental principles of expressions and the permanence of form (the idea of expression)
4. Clarifying and extending the meaning of things and operations, and attempting to think based on this (the idea of operation)
5. Attempting to formalize operation methods (the idea of algorithms)
6. Attempting to grasp the big picture of objects and operations, and using the result of this understanding (the idea of approximation)
7. Focusing on basic rules and properties (the idea of fundamental properties)
8. Attempting to focus on what is determined by one's decisions, finding rules of relationships between variables, and using relationship (functional thinking)
9. Attempting to express propositions and relationships as formulas, and to read their meaning (the idea of formulas)

Source: (Isoda & Katagiri, 2012, 2016)

Shigeo Katagiri (Katagiri, Sakurai & Takahashi, 1969; Katagiri *et al.*, 1971), a curriculum specialist in primary school mathematics in the Ministry of Education, established the framework for mathematical thinking with teachers, Figure 11. He published 40 guidebooks for teachers with his LS group.

The Framework listed in Figure 11 is used in LS to develop mathematics problem for knowing which type of mathematical thinking is necessary to use for solving a problem. And then, it is used with the purposes of clarifying in a lesson plan what kinds of questions are necessary to pose to promote the mathematical thinking. It does not correspond to a list of hints such as the strategies for solving problems adapted from Polya (1945). It also explains the needs to practice like the recommendations in the list. It is used for writing objectives more concretely and to clearly specify the teaching with each material and process. It also provides the general study subject/topic of the lesson study beyond every objective of the teaching content. Katagiri also developed the list for questioning in the classroom in relation to teaching phases.

5.1. Task Sequence to Develop Mathematical Thinking

The Middle School at Tokyo Higher Normal School, after WWII, was divided into Junior High School and High School at Tokyo University of Education. Both mathematics teachers' groups were active in the already mentioned Union's group at the beginning stage. However, they became independent and established their own LS groups because they did not agree with Toyama's theory and leadership, and they oriented more freely discussion for mathematics curriculum development by themselves. Kiyoshi Yokochi established Mathematics Education Society of Japan (not JSME) at the High School. Toshio Odaka established School Mathematics Study Society at the Junior Secondary School of the Tokyo University of Education- SMSS-JSS-TU. He developed his own schema theories (Odaka, 1975, 1979, 1980) for a problem-solving approach (SMSS-JSS-TU, 1971, 1972), inspired by the idea of Piaget for supporting the extension and integration principle from the tradition of mathematization in the 1943 national textbook, as its successors.

Odaka's theory, called the "exemplar approach", was a counter theory to explain an appropriate curriculum and a task sequence, with his own schema theory under Piaget and against Toyama's one. Exemplar means the task (problem) which represents each schema, and his task sequence for problem solving approach (Odaka & Okamoto, 1982) is explained by the sequence of schema in which all necessary schemas explain what students shall learn at primary and junior high schools. Figure 12 is an exemplar problem and its renovation.

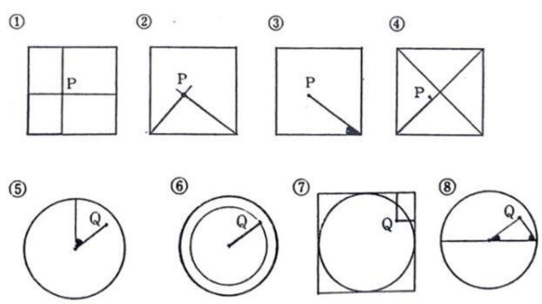
Figure 12: How can we introduce axis?

For the Introduction of Axis

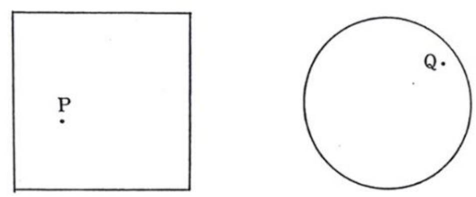
From Odaka *et al*. (1970):

There are a square (1) and a circle (2), its side is 6cm and its radius is 6 cm, respectively. Let's consider the ways to explain the positions of P in the square and Q in the circle.

Possible solutions:



(1) (2)



Objective:

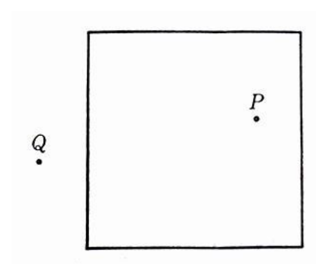
To produce necessary and sufficient conditions to set the positions of the points by using given lines and point on the plane and naming A, B, AB and so on.

From Odaka *et al*. (1982):

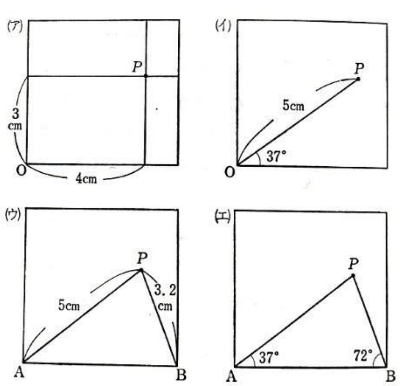
There is a 6 cm-side square. P is inside and Q is outside of it.

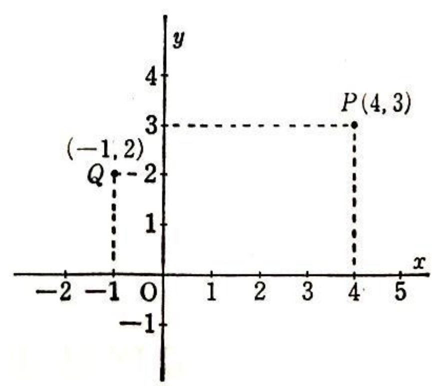
1) Let's consider the ways how to explain the position of P.

2) Let's consider the ways how to explain the position of Q.



Possible solutions for 1): Final conclusion after 2)





Objective (Schema): The position is represented by the axis.

Source: Odaka et al (1970) and Odaka et al (1982)

On Figure 12, Odaka's group tried to realize the idea of Figure 8 of the integration of geometry and algebra, based on their LS. The task sequences of Odaka 1970 and 1982 are not the same either the objective is not the same. Odaka 1970 only asked the positions of points inside of figure. Their discussion focused more on given conditions such as lines (sides) on (1) and point (center) on (2). After several LS, they revised the task sequence for the introduction of Axes, like Odaka 1982. Firstly, students think various possible ways to represent an inside point P. Secondly, they apply their ideas to Q and finally they conclude that extending the segments to the lines are the most simple and sufficient way: It is Cartesian Axis. At the same time, students could discuss further ideas for unknown System of Axis such as for complex plane.

This interesting Task of 1982 in Figure 12 was also embedded into the student's textbook under National Standards. However, the sales were not good because from the perspective to learn the algebraic operation of functions, teachers would prefer to teach function right after Algebra, and not after learning both Algebra and Geometry. For new generation of teachers, the order Algebra, Function and Geometry is felt better to enrich students' algebraic skills and they did not know the historical discussion of the integration of subjects on Figure 8.

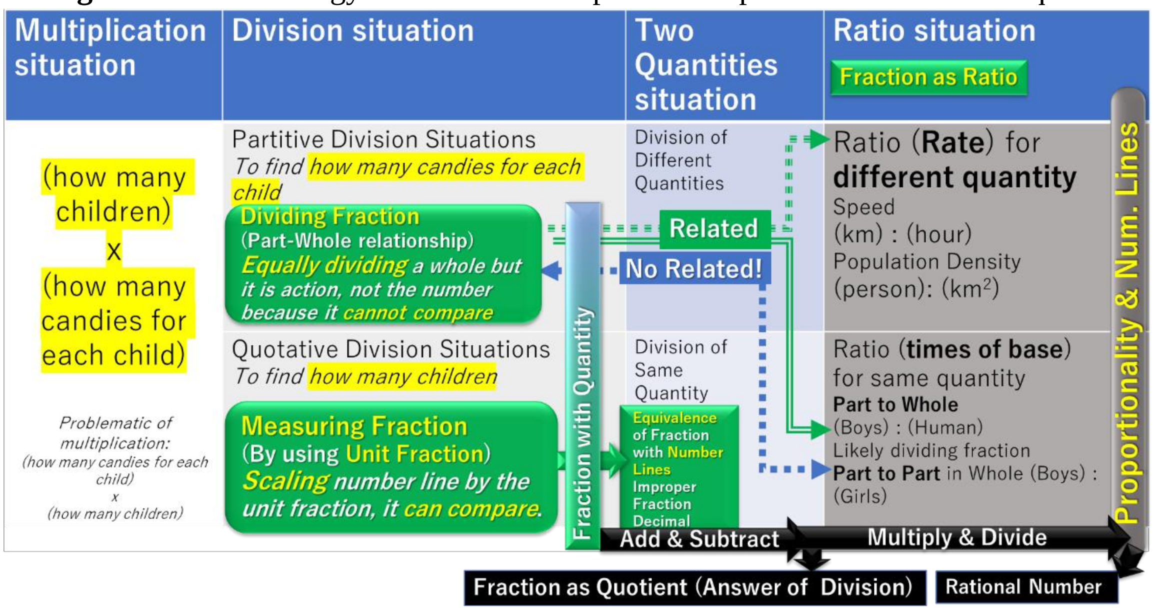
5.2. Terminology for Explaining Curriculum and Task Sequence

Japanese LS produced necessary Terminology which distinguishes conceptual differences and its sequential development in curriculum. It is part of the theory for mathematics education at primary school with LS. It includes the technical terms to distinguish conceptual differences such as different meaning of multiplication and the representations such as Tape Diagram and Proportional Number Lines for overcoming such differences. It is necessary to explain the process of reorganization of concepts in the curriculum sequence. The Japanese established most of it between 1900 and the 1960s, as stated in the special issues of the Journal of Mathematics Education published by the Japan Society of Mathematical Education, in 2010.

Through school-based LS, Japanese teachers learn it to explain why they planned such a task and questions for developing students who learn mathematics by and for themselves. Even when National Standards indicate the content what they should teach, it does not explain well why they have to teach it in his/her order of task-sequence. For developing and promoting students thinking, teacher must use what students already learned.

Figure 13 is a Terminology in the case of Fraction. If you identify any unknown words, it is a terminology which was established for LS (Isoda, 2021). Terminology makes possible to distinguish conceptual differences and its sequence. Odaka's Schema theory has also oriented such terminology.

Figure 13. Terminology for Fraction to explain conceptual difference and sequence.



Source: (Isoda, 2021)
Isoda, M.: Fraction for Teachers: CRICED, University of Tsukuba revised.
https://www.criced.tsukuba.ac.jp/en/pdf/Fraction_for_Teachers_(full_version).pdf

The principle of the Japanese curriculum standards "extension and integration" is oriented toward enhancing mathematical activities and developing mathematical thinking. It corresponds to the principle of reinvention by Freudenthal (1973) who proposed the mathematization as the reorganization of mathematical experience (Isoda, 2018).

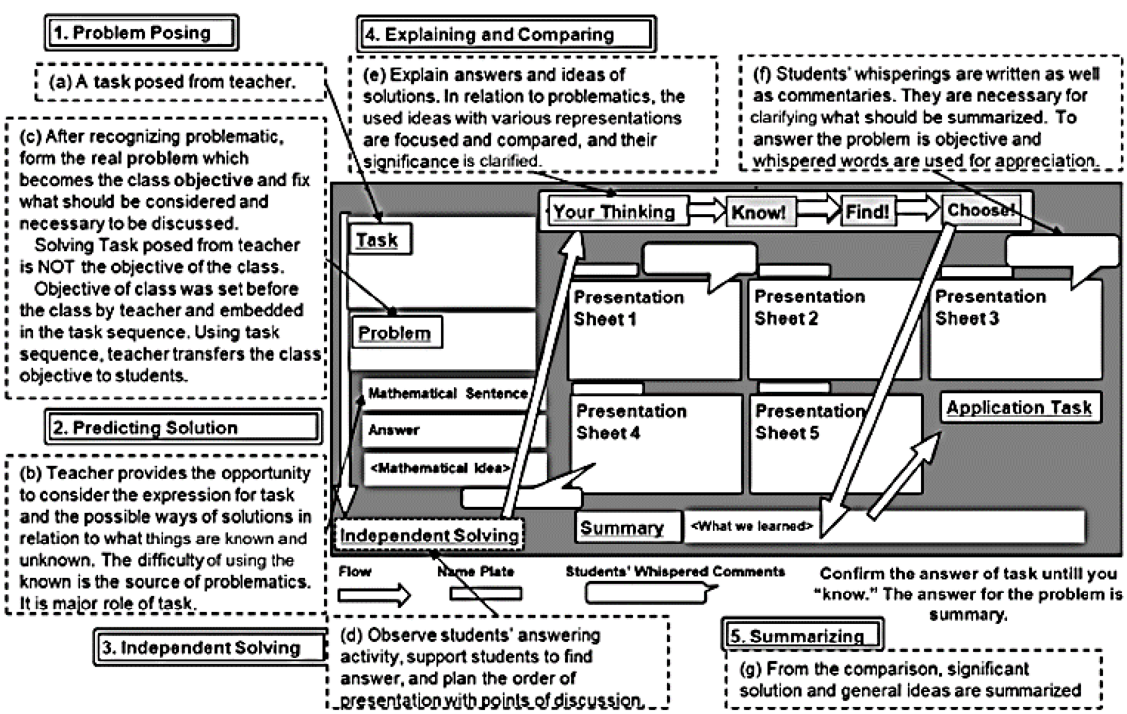
Under this principle, the school mathematics curriculum can be seen as a set of partially ordered local mathematics theories, like a net that is consistent within every local theory like a knot. However, on extending and integrating local theories, the net has some inconsistencies in connecting the local theories, like entangled strings among knots of Figure 10. Besides such inconsistencies, Mathematical thinking framework by Katagiri enhanced what way of thinking, attitude and values should be necessary to be consistently developed beyond the contradictions. This means that the mathematical thinking framework functions as the Mathematical Literacy Framework.

6. Teaching through Problem Solving: Curriculum implementation

Japanese Problem-Solving Approach (PSA) is known as teaching mathematics through problem-solving. Open Approach by Nohda, N (2000) and Open-Ended Approach by Becker and Shimada (1997, in Japanese 1977) are also known as a part of PSA. Those approaches are product of LS by teachers who have tried to teach mathematics through mathematical arguments such as dialectic by students, through generations. Indeed, Shimada is one of contributors for Figure 8, before WWII. It is the product of LS before WWII with the efforts of the schools, that the current practice of PSA at each school-based LS can be seen everywhere in Japan, since 1980s. Narrow meaning of PSA is distinguished with Open Approach because PSA is exactly planned to reorganize mathematics by using what students already learned before, under the curriculum and through a task sequence.

PSA distinguished Task, which is given by teacher, from the Problematic, which is recognized by students as unknown. Problematic, in other words, constitutes difficulty and challenges for students, and it is planned by the class-teacher based on the content unit and yearly plan through the task sequence, so that overcoming problematic itself is an objective of the planned lesson. Figure 14 is a sample board that utilizes the format of PSA.

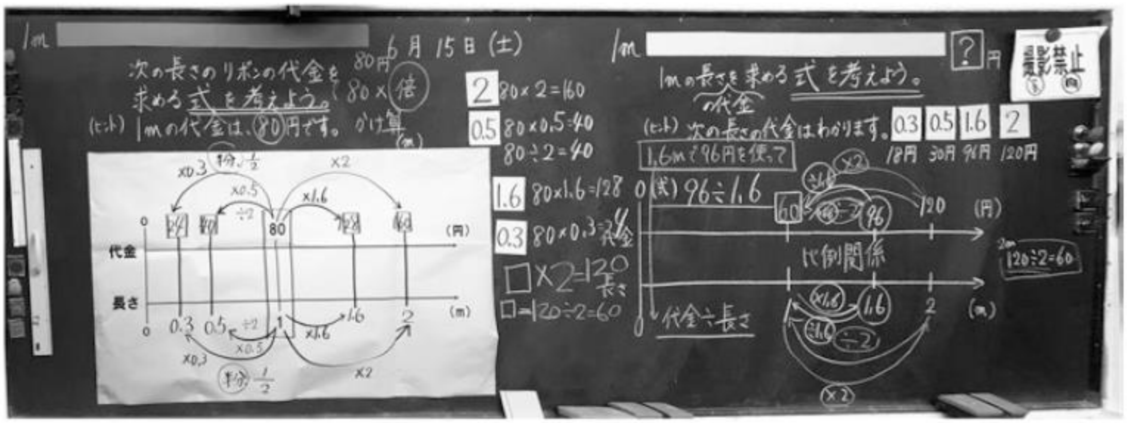
Figure 14. A board registering the lesson plan: teacher never erases the board for reflection.



Source: (Isoda & Katagiri, 2012, p. 16; 2016 Spanish Edition)

Figure 15 corresponds to the case of proportionality by Seiyama, that uses two meanings of the division. Seiyama is well known teacher through the JICA -LS video on YouTube which has various language subtitles.

Figure 15. LS utilizing Proportional Number Lines since 1960s. (Seiyama open-class-2019).



Source: (Isoda & Olfos, 2021, p. 95)

7. Final Consideration: Next Steps as International Challenges

Japanese PSA is known internationally as a model approach to LS. However, for many countries it is not easy to demonstrate such a lesson, even if it is possible to implement Open Approach for LS. Open Approach lessons demonstrate collaborative learning, so it is good to develop learning community. However, it is rare to see the argumentations/dialectic amongst students which is the real authentic mathematical activity. Major difference is that Open Approach focus on solving a given task itself, even if people call it a problem situation. On PSA, students are urged to solve problematic situation for them, and on the way of solving it as objective they become able to engage in mathematical arguments. Even in the case a task is given, a problematic with the search for unknown should be posed by students.

For challenging PSA, we need correct and adequate *Terminology, Task Sequence*, and the *Necessity to solve the task* based on what students already learned. And as choosing a preferred PSA, teacher must explain clearly his/her objective of why he/she chose it. On this demand, Isoda developed the curriculum framework for Southeast Asia, the teacher training books for problem solving, and has adapted textbooks in Thailand, Indonesia (National) and Chile (National) with collaborations of leading researchers and ministries, he and engages in LS.

Especially under Asian Pacific Economic Cooperation (APEC) LS Network and Southeast Asia Ministers of Education Organization (SEAMEO) School Network, the Center for Research on International Cooperation in Educational Development (CRICED), University of Tsukuba, Japan has recently opened online courses for teacher education on YouTube which already registered more than 10,000 users from 38 countries (http://criced.tsukuba.ac.jp/en). Establishment of LS community in Brazil using these tools is a big dream which will be realized through the collaborations.

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Authors
Masami Isoda
Bachelors in mathematics. University of Tsukuba
Master in education. University of Tsukuba
Ph.D. in Education: Waseda University
University of Tsukuba
CRICED, Lesson Study
isodamasami@yahoo.co.jp
https://orcid.org/ 0000-0002-0017-3935

Yuriko Yamamoto Baldin
Bacharelado e Licenciatura em Matemática pelo IMECC-UNICAMP
Mestre em Matemática pelo IMECC-UNICAMP
Doutor em Matemática pelo IMECC-UNICAMP
Departamento de Matemática/CCET/UFSCar
GIPEM- Formação de Professores, Lesson Study, Materiais didáticos/Tecnologia/Ensino
yuriko@ufscar.br
https://orcid.org/0000-0001-7473-5657

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