The Lesson Study in the context of PIBID: challenges and possibilities for the initial teacher education of Brazilian Mathematics teachers

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Abstract

In this research, with a qualitative approach of the interpretative type, we aim to answer the following question: What are the limits and potentialities regarding the theoretical and practical aspects of lesson study in the initial formation of mathematics teachers in the context of PIBID? Ten (10) mathematics graduates who, during a period of PIBID, experienced the Lesson Study in the study/research, lesson planning, execution and post-class reflection about the object of second-degree equation knowledge participated in the research. For the production of the data, we used participant observation, field notes of the researchers, audio and video recordings of the meetings held on the Microsoft Teams platform and also reports prepared by these participants. Through the analysis and discussion of the data, these data categorized and interpreted using the typology of teaching knowledge proposed by Shulman, we conclude that the Lesson Study enabled, to the Initial Formation of these graduates, the learning of specific and didactic knowledge of the content related to the object of mathematical knowledge mentioned above, in addition to knowledge of the general pedagogical type, the student and its characteristics and educational contexts. The limits refer to the impossibility of being present in the preparation and execution of tasks due to the Pandemic period.

Keywords: Mathematics Teacher Education. Lesson Study. Teaching Knowledge.

Un estudio de clase en el contexto de PIBID: desafíos y posibilidades para la Formación Inicial de profesores brasileños de Matemáticas

Resumen

En esta investigación, con un enfoque cualitativo de tipo interpretativo, pretendemos responder a la siguiente pregunta: ¿Cuáles son los límites y potencialidades con respecto a los aspectos teóricos y prácticos del estudio de la lección en la formación inicial de profesores de matemáticas en el contexto de PIBID? Diez (10) graduados en matemáticas que, durante un período de PIBID, experimentaron el Estudio de la Lección en el estudio/investigación,

planificación de lecciones, ejecución y reflexión post-clase sobre el objeto de conocimiento de ecuaciones de segundo grado participaron en la investigación. Para la producción de los datos, se utilizó la observación participante, notas de campo de los investigadores, grabaciones de audio y video de las reuniones realizadas en la plataforma Microsoft Teams y también informes preparados por estos participantes. A través del análisis y discusión de los datos, estos datos categorizados e interpretados utilizando la tipología de conocimiento docente propuesta por Shulman, concluimos que el Estudio de la Lección permitió, a la Formación Inicial de estos graduados, el aprendizaje de conocimientos específicos y didácticos del contenido relacionado con el objeto de conocimiento matemático mencionado anteriormente, además de conocimientos del tipo pedagógico general, del alumno y sus características y contextos educativos. Los límites se refieren a la imposibilidad de estar presente en la preparación y ejecución de tareas debido al período de Pandemia.
Palabras clave: Formación de profesores de Matemáticas. Estudio de Clase. Enseñanza del Conocimiento.

A *Lesson Study* no contexto do PIBID: desafios e possibilidades à Formação Inicial de professores de Matemática brasileiros

Resumo

Na presente pesquisa, de abordagem qualitativa do tipo interpretativa, visamos responder ao seguinte questionamento: Quais os limites e as potencialidades a respeito dos aspectos teórico-práticos da *Lesson Study* na Formação Inicial de professores de Matemática no contexto do PIBID? Participaram da investigação 10 (dez) licenciandos em Matemática que, durante um período do PIBID, experienciaram a *Lesson Study* no estudo/pesquisa, planejamento de aula, execução e reflexão pós-aula a respeito do objeto do conhecimento equação do segundo grau. Para a produção dos dados, utilizamos a observação participante, notas de campo dos pesquisadores, gravações em áudio e vídeo dos encontros realizados na plataforma da *Microsoft Teams* e, ainda, relatórios elaborados pelos referidos participantes. Mediante a análise e a discussão dos dados, dados esses categorizados e interpretados valendo-se da tipologia dos conhecimentos docentes proposta por Shulman, concluímos que a *Lesson Study* possibilitou, à Formação Inicial desses licenciandos, a aprendizagem de conhecimentos específico e didático do conteúdo referentes ao objeto do conhecimento matemático supracitado, além de conhecimentos do tipo pedagógico geral, do aluno e de suas características e dos contextos educacionais. Já os limites referem-se à impossibilidade de estarem presentes no preparo e na execução das tarefas em virtude do período Pandêmico.
Palavras chave: Formação de professores de Matemática. Estudo de Aula. Conhecimentos Docentes.

Introduction

In this paper we present the results of a formative process - inserted in the context of the Institutional Program of Scholarship Initiation to Teaching (PIBID) - carried out through Lesson Study aimed at the initial training of mathematics teachers. Among the various challenges observed when we problematize and act upon this training, we can mention the need for academics, future teachers, to learn-build a wide range of specific knowledge for teaching, as

already pointed out by Shulman (1986, 1987), as well as the (re)signification/re-education of conceptions that must occur during the undergraduate course. These conceptions have already been constructed during elementary education and refer to mathematics and its teaching-learning process, as observed in some studies (FIORENTINI et al., 1998; LÓPEZ; ALSINA, 2016; SILVA; PASSOS, 2016; SAKAY, 2007).

In line with Nascimento, Carvalho, and Ramos (2022), we chose the construction of the lesson as the leitmotif of this training because we consider it the starting point of the constitutive process of being a teacher. And in this sense, the study of the lesson, as pointed out by several authors (BEZERRA, 2017; BURROUGHS; LUEBECK, 2010; ESTRELLA; MENA-LORCA; OLFOS, 2018; FERNÁNDEZ, 2010; FERNÁNDEZ; ZILLIOX, 2011; FUJII, 2018; ISODA; ARCAVI; LORCA, 2007; PINA NEVES; FIORENTINI, 2021; PONTE; ISODA; OLFOS, 2009; RICHIT; PONTE; TOMKELSKI, 2019; PONTE; WAKE; QUARESMA, 2020; STIGLER; HIEBERT, 1999; VIEIRA; PONTE; MATA-PEREIRA, 2022), is suitable for this objective because it consists in the development of a lesson, starting from a problem arising from the teaching practice, "going through" the phases of study, planning, realization and post-performance reflection of the lesson, always with the aim of the students' learning.

In this scenario, we have as a research problem the following question What are the limits and potentials regarding the theoretical and practical aspects of Lesson Study in the initial training of mathematics teachers in the context of PIBID? In the next sections, we will discuss initial teacher education and teaching knowledge from the perspective of Shulman (1986, 1987), followed by a brief contextualization of the PIBID, a program that represents an important public policy of valorization and incentive for teaching in Brazil. Next, we present the PIBID/Mathematics - Foz do Iguaçu, which constitutes the formative space in which the present research was carried out. After that, we will situate Lesson Study historically, as well as its defining elements, and briefly discuss its use in/for teacher education. Next, we present the methodology of our research, followed by the analysis and discussion of the data that will allow us to answer our research problem. Finally, we bring the final considerations observing the contributions and implications to the field of mathematics education.

Initial Teacher Education and Teacher Knowledge

Teacher education, as emphasized by García (1999, p. 26), constitutes a "[...] field of knowledge, research and theoretical and practical proposals that [...] studies the processes through which teachers - in training or in practice - engage individually or in teams in learning experiences". In our case, we are looking at the stage "in formation", or also called Initial Formation, which takes place during the course of a bachelor's degree in mathematics.

With regard to initial education, Mizukami et al. (2006, p. 22) point out its importance in providing "[...] good support to prepare them [teachers] to act in the profession". In the same sense, Imbernón (2000) understands that initial training should be the basis for the constitution of specialized pedagogical knowledge, being the beginning of professional socialization and the adoption of principles and practical rules.

The research on this specialized pedagogical knowledge, or the specific knowledge and expertise necessary for the exercise of teaching, as well as the thinking/questioning about it mobilized by the teacher before/during/in the classroom (and school) action, has generated a significant amount of research aimed at classifying/categorizing such "knowledge and expertise" according to epistemological, psychological, practical-phenomenological, etc. criteria. Such categorizations are commonly referred to in educational literature as typologies of teacher knowledge and expertise. Among the various typologies (FREIRE, 2003; GAUTHIER et al., 1998; PACHECO; FLORES, 1999; PERRENOUD, 2000; PIMENTA, 2008; PORLÁN; RIVERO, 1998; TARDIF, 2003), we chose the one synthesized by Shulman (1986, 1987).

In his analysis of the research programs and paradigms "process-product" and "teacher thinking", Shulman (1986) states that research, in an attempt to simplify the complexities, present in the classroom, ignores a central aspect of teaching: the specific content of the discipline that the teacher teaches. According to the author, such research has not been examined:

> [...] how the specific content of a knowledge area was transformed based on the knowledge that the teacher had into teaching knowhow. Nor did they ask how particular formulations of content related to what students came to know or mislearn (SHULMAN, 1986, p. 6).

In this sense and aiming to contribute to the construction of a solid body of knowledge for teaching, Shulman (1987) develops a research program, the knowledge base (knowledge

base for teaching). This 'base', according to the author, is gradually built from four sources: a) academic knowledge of subject content (to teach); b) educational structures and materials; c) academic knowledge of formal education (the systematized pedagogical knowledge in its various dimensions: psychological, philosophical, curricular, sociological, historical, methodological, etc.) and; d) wisdom of practice (the least codified source of all).

Table 1, in the direction of indicating the referred 'base', presents the diverse knowledge identified by the referred author:

Table 1 - Shulman's Typology of Teacher Knowledge

Teaching Skills "Knowledge"	**Definition**	**Knowledge - Social Sources of Acquisition**	**Modes of Knowledge Integration in Teacher Education**
1. Teaching Content Knowledge (Specific)	These are the teacher's understandings about the structure of the subject, how he/she cognitively organizes the knowledge of the subject to be taught. To understand the structure of a discipline, the teacher needs to know - in addition to facts and concepts intrinsic to it (substantive knowledge) - the ways in which the fundamental principles of an area are organized, the processes of its production, representation and epistemological validation (syntactic knowledge).	Formal and academic contact in the area of specialty sciences (Mathematics, Physics, Chemistry, Biology, Geography, etc.) during the course (Licentiate) in Higher Education. The accumulated literature and studies in the content areas.	By the training and socialization in the course (Licentiate) about this knowledge coming from the specialty sciences.
2. General Pedagogical Knowledge	This is knowledge that transcends mastery of a specific area, including knowledge of educational goals, objectives, and purposes. It also refers to the 'management' of the class and interaction with students, the way students learn, instructional strategies, teaching and learning process.	The formal contact during the course (Licentiate) with the academic knowledge of formal education. The bibliographical documentation and research.	By the training and socialization in the course (Licentiate) about the academic knowledge of formal education. By the investigation arising from the pedagogical practice.
3. Pedagogical Content Knowledge	These are the ways of formulating and presenting content in a way that makes it understandable to students, including examples, illustrations, analogies, demonstrations, explanations, etc. Included is the teacher's understanding of what facilitates or hinders the learning of a given content, as well as the students' misconceptions and their implications on learning. It is the only knowledge that the teacher is the protagonist, because it comes from his or her professional performance.	Pedagogical practice. The literature - research - about case study(s).	By pedagogical practice. By the literature study - research - about case study(s).

4. Knowledge of the Curriculum	Represents the curriculum, that is, the set of programs designed for teaching specific subjects at a given level (Kindergarten, Elementary/Middle School, Technical School, Higher Education). Also included are the instructional materials available for the programs.	The use of programs, textbooks, instructional materials. The literature - research - on this topic.	By the use of the programs, textbooks, instructional materials.
5. Knowledge of the Students and their Characteristics	It refers to knowledge that is specific to a given context (classroom) and is different (changeable) for each.	Pedagogical practice.	By pedagogical practice.
6. Knowledge of Educational Contexts	This is the knowledge that comes from group or classroom work. This knowledge varies according to the government, cultural aspects, and characteristics of the community in which the school is located.	Pedagogical practice. The school and its social (cultural) context.	By pedagogical practice. By the experience derived from living in a certain social (cultural) context.
7. Knowledge of Educational Aims, Purposes and Values	It refers to the understanding of how the historical-philosophical aspects - developed throughout history - articulate with the purposes, objectives, and values that were, and are being, defined and adopted by the educational institutions according to the social and cultural moment.	Pedagogical practice. Society in its social, cultural, economic aspects.	By pedagogical practice. By the experience derived from social coexistence.

Source: Elaborated from Shulman (1986, 1987)

Thus, in order to collaborate in such a way as to provide good support to the Initial Formation of (Mathematics) teachers, and to contribute to the constitution of specialized pedagogical knowledge, the PIBID 'arises', as presented below.

The PIBID project: a public policy for teacher education

The Institutional Teaching Initiation Scholarship Program (PIBID) was created by Decree No. 7.219/2010 (BRASIL, 2010) and regulated by Ordinance 096/2013 (BRASIL, 2013), in which the Coordination for the Improvement of Higher Education Personnel (CAPES) is in charge. It is a public policy that aims to encourage the Initial Training of teachers, and, to this end, it brings the university closer to the school and articulates Initial Training with Continuing Training, in a continuum that aims at the appreciation of the teaching profession.

> [...] and as important as the other purposes, come two aspects: to encourage teachers who graduate to participate more actively in the life of schools at this level of education, getting to know them "inside", and to teachers at basic school to take responsibility for the training of future teachers (TANCREDI, 2013, p. 14).

According to the CAPES website (http://www.capes.gov.br/educacao-basica/capespibid/pibid), the PIBID has the following guiding objectives:

- encourage the training of teachers for Basic Education;
- valuing the teaching career;
- improving the Initial Formation of teachers in the undergraduate programs, through the integration between Higher Education and Basic Education;
- to insert the undergraduate students in the daily life of the school, promoting methodological, technological, and practical experiences that aim to overcome the difficulties identified in the teaching-learning process;
- to encourage public Basic Education schools, through their teachers, to assume the role of co-trainers of future teachers, making them protagonists in the Initial Training processes for teaching; and
- to articulate the theory and practice needed for the formation of teachers, promoting the experience of actions in the Undergraduate courses.

The program is made up of an institutional coordinator (university teacher), an area coordinator (university teacher), a school supervisor (basic education teacher) and novice teachers (undergraduate students). All these functions are paid for with scholarships, but the project also counts on the participation of volunteer teachers and students.

Through the Edictal nº 02/2020 (BRASIL, 2020), CAPES selected proposals from all over Brazil, including the Universidade Estadual do Oeste do Paraná (UNIOESTE). The Faculty of Mathematics of Foz do Iguaçu, together with two other faculties, Mathematics of Cascavel and Chemistry of Toledo, proposed the Interdisciplinary Project Mathematics/Chemistry, with 24 scholarship students, 3 basic education supervisors with a scholarship, a university project coordinator with a scholarship, and also six academics (two from each campus) and five university teachers (from the three campuses) as volunteers.

The activities of the PIBID began in the middle of the COVID-19 pandemic, when teaching, both in higher education and basic education, underwent a profound restructuring due to the physical and social isolation imposed on us all. In this scenario, in addition to the challenges already inherent in initial teacher education (the difficulty of integrating specific and

pedagogical subjects, the conceptual discrepancies of the students coming from basic education, the re-education of the conceptions already built by the students about mathematics and its teaching and learning, the extensive "load" of subjects to be accomplished, etc.), in this version of PIBID we also had the problem of the pandemic. Thus, it was necessary to rethink the program, in the midst of rethinking teacher education as a whole, in a context of uncertainties and difficulties generated by physical and social isolation and the use of technology.

In order to make the PIBID/Mathematics - Foz do Iguaçu possible, it was necessary to think of strategies that would motivate the student (future mathematics teacher), that would be feasible and that would also promote discussions and reflections uniting the Initial and Continuing Education of teachers. Based on this, we chose to work with the Lesson Study, because it allows the union of theory and practice, the work with a real classroom problem, and also to articulate the Initial and Continuing Education, which was the motivation and guide for the development of the work done with the "pibidians".

In the next topic, we present and discuss what it is and how we understand the Lesson Study used in the PIBID/Mathematics - Foz do Iguaçu[1] in the year 2021/2022.

A Lesson Study: a possibility for Teacher Education

Lesson Study (or Estudos de Aula/Estudos de Lição in Portugal or Estudio de Clases in Spain) is a formative process based on reflection and collaboration. It appeared in Japan at the end of the 19th and beginning of the 20th century under the name Jyugyo Kenkyu. Since then, it has been used in the Japanese school culture and is divided into three stages: a) planning (in which a lesson is structured); b) development (in which the lesson is carried out as planned); c) analysis (moment in which the lesson planning and execution are analyzed).

According to Soto Gómez and Pérez Gómez (2015), lesson study brings principles from action research, Pestalozzi's philosophy, and Dewey's experiential learning. Ponte et al. (2016) point out the possibility of joint work between initial and continuing teacher education, based on the training context made possible by Lesson Study; however, these works are still rare in Brazil, and even smaller in relation to initial education. According to Pina Neves, Fiorentini and Silva (2022):

[1] From this point on, to refer to the "PIBID/Mathematics - Foz do Iguaçu", only the word PIBID will be used in the body of the text.

> In Brazil, the first studies in LS were mostly conducted in the Southeast Region and in continuing education contexts [...] it is observed that the number of studies conducted in other regions has been growing, at the same time as the interest for the development of LS in initial education [...] (p. 417).

The first lesson study studies in Brazil are still recent, as evidenced by the work of Baldin (2009), Felix (2010), Coelho (2014), Merichelli and Curi (2016), Bezerra (2017), Fiorentini et al. (2018), Crecci et al. (2019), Richit et al. (2019), Wanderley and Souza (2020), Macedo, Bellemain and Winslow (2020), Pina Neves and Fiorentini (2021), among others, which are prevalent in the professional development of teachers who teach mathematics.

Throughout the world, Lesson Study is being implemented with adaptations based on what has been proposed in Japan. These adaptations are due, among other things, to the need to adapt this educational process to local realities. For example, in Brazil, due to its large geographical size, the adaptations have observed regional, cultural and economic differences.

In the present study, we chose to make an adaptation of the methodological proposal of Lesson Study, used in Continuing Education by Bezerra (2017) and synthesized by Bezerra (2020), which consists of:

STAGE 1: Planning - the moment when the group collectively chooses the object of mathematical knowledge and the objective for the preparation of the lesson. This stage is divided into several phases, namely a) choice of the subject of mathematical knowledge; b) research on the chosen subject of mathematical knowledge (in official documents, textbooks, paradidactic books and others); c) exchange of experience on the approach to the chosen subject of knowledge by the members of the group, either in the condition of students or in the condition of teachers; d) the members of the group individually and then jointly think about and select challenging problem situations for teaching the chosen object of mathematical knowledge; e) the group tries to anticipate the students' reasoning in each of the elaborated problem situations, discussing the degree of difficulty and possible changes; f) the group solves the elaborated problem situations and, if necessary, reformulates them.

STEP 2: **Conducting the Class** - The class is conducted by one of the members of the group and observed by the others. The lesson is recorded by means of video and individual reports to support later reflections.

STAGE 3: **Reflection** - After the lesson has taken place, the group analyzes the lesson by watching the recording and reading the individual reports. Through the discussions, it is possible to reflect on the lesson developed by the group, evaluate the difficulties in the teaching and learning process, and suggest changes for the lesson.

Thus, as already highlighted, the class - and the problems arising from it - constitutes the starting point of this formative context made possible by Lesson Study (BEZERRA, 2017; BURROUGHS; LUEBECK, 2010; ESTRELLA; MENA-LORCA; OLFOS, 2018; FERNÁNDEZ, 2010; FERNÁNDEZ; ZILLIOX, 2011; FUJII, 2018; ISODA; ARCAVI; LORCA, 2007; PINA NEVES; FIORENTINI, 2021; PONTE; WAKE; QUARESMA, 2020; PONTE; ISODA; OLFOS, 2009; RICHIT; PONTE; TOMKELSKI, 2019; STIGLER; HIEBERT, 1999; VIEIRA; PONTE; MATA-PEREIRA, 2022).

It is also important to note that Lesson Study is grounded in the perspective of collaboration. Although the meaning of collaboration is polysemic, we adopted in this research the meaning attributed to Boavida (2005). According to the author

> The paths of collaboration can be very different. However, for a research project to be successful it seems essential to have voluntary participation, to negotiate carefully, honestly, openly and from the beginning how the group will function, to outline a path of working together that is understood to be appropriate and feasible given everyone's needs, goals, interests, expectations and desires, and to negotiate and renegotiate each other's responsibilities and roles so that the benefits of complementary experiences, perspectives and skills govern the collaborative process for all involved. Mutuality in the sharing of goals, roles, and responsibilities is not essential. What is important is that there is a common purpose that guides the work to be developed, but also the possibility of particular interests and needs, that there is shared leadership and a non-hierarchical relationship, and that a climate of ease and mutual respect is created so that each person can share knowledge and experience, and, in this way, everyone can learn (BOAVIDA, 2005, p. 191).

Therefore, trust (expressing oneself without fear of making a mistake), conversation and negotiation are necessary elements for collaboration and should be present in the Lesson Study considered as a formative context in/for the Initial and Continuing Education of the teacher who teaches Mathematics.

Research Methodology

From our question, "What are the limits and potentials regarding the theoretical and practical aspects of Lesson Study in the initial training of mathematics teachers in the context of PIBID?", we chose the qualitative research approach (MINAYO, 2011), of the interpretive type (ERICKSON, 1986), in which we seek plurality in the interpretation and understanding of contexts, as suggested by Sandín Esteban (2010).

The students who were members of the PIBID, as well as the mathematics teacher who supervised the project and worked at the school, were informed that the PIBID meetings were

also part of a research project approved by the Ethics Committee (CAAE: 47772921.0.0000.0107), and therefore they were asked to sign the informed consent form.

For the production of data, we used participant observation, researchers' field notes, audio and video recordings, considering that all the meetings were held through the Microsoft Teams platform, and also the reports written by 10 (ten) “pibidans” (eight of them fellows and two volunteers) and posted on the aforementioned platform.

Participant observation was used because it is a "[...] strategy that involves, [...] not only direct observation, but a whole set of methodological techniques that assume a great involvement of the researcher in the situation studied" (LÜDKE; ANDRÉ, 1986, p. 28), so that one can hear, listen, see, use all the senses, being aware that one is in the group, but being someone "outside" the group, as pointed out by Whyte (2005).

The audio and video recordings offered us "[...] a limited but powerful record of temporal actions and real - concrete material events" (BAUER; GASKELL, 2015, p. 137), and this was valuable, especially considering the pandemic period we lived in during the course of the PIBID.

And finally, the reports created by the “PIBIDans” and posted on the Microsoft Teams platform had the function of a narrative in which, through writing and (re)constructing events, as suggested by Bauer and Gaskell (2015), they had the opportunity to relive the experience and find possible explanations for what happened.

The activities were developed over five months of the year 2021, in weekly meetings, as described in Table 2.

Table 2 - Activities developed in the PIBID Math/Foz do Iguaçu Project through the Lesson Study training context

Day/Month	Activities developed
Initial Motivation	
30/03	The supervisor of the school, a mathematics teacher, presented to the PIBID group the difficulty in teaching and having the students understand the content "Second Degree Equation" when linked to a context.
Lesson Study Context	
06/04	On this day, the Lesson Study was presented to the group as a way to motivate them to use this context to assist the supervising teacher at the school in teaching the content "Second Degree Equation".
13/04	The students watched six videos on the YouTube channel "Lesson Study - Maria Alice" about what Lesson Study is and how it works. (https://www.youtube.com/channel/UC5XJsNGUlzmvPoVsFW3ooLw)
20/04	Discussion of the videos watched on 4/13, the historical context in which Lesson Study emerged in Japan, and how this formative context has been used around the world and in Brazil.
Studying and Researching the Content	
04/05	The future mathematics teachers were asked to formulate and present their concepts about the content of the second-degree equation. The challenge was that the concept formulated should

	express the understanding of the content without using the mathematical concept established in the textbooks.
11/05	From the presentations, the group decided that it was necessary to understand and define the concept of function in order to also define and understand the concept of equation and, subsequently, the concept of second-degree equation. Furthermore, it was suggested by one of the future teachers and accepted by the group the idea that history would be an important resource for this understanding, as it would help to understand how the contents 'appeared' and were formalized over time.
25/05	Each pibidian student presented the information found throughout the historical research. After each presentation, the group discussed and complemented the information.
01/06	Continuation of the presentations and discussion. At the end of the presentations, it was discussed that history can be motivating, can be just informative, and can be used as a resource for "new" discoveries. In view of this, the reading of three texts was suggested, namely: "Three Studies on History and Mathematics Education", by Miguel (1993); "A little History of Functions: some suggestions for practical activities for the classroom", by Oliveira, Viana and Rosa (2013) and "The History of the Concept of Function in Video: a proposal for learning", by Maciel and Cardoso (2014).
15/06	The texts read were discussed and the students were asked to review the definitions of function, equation and second-degree equation formulated individually from the discussions and reflections held in and with the group.
22/06	The objective of this meeting was, based on the individual definitions, to create a collective definition of the concept of function, equation, and second-degree equation. The discussion was long and full of comings and goings, because the goal was for the group to understand the differences and formalize a concept without the need to look for the concepts elaborated in the textbooks. For the next meeting, the students were challenged to think of problems (elaborate or re-elaborate) that use the second-degree equation content to work (conduct a lesson) with ninth graders as requested by the school's supervising teacher.
Preparation of the Lesson	
29/06	Presentation and discussion of the problems prepared/reworked by each of the students for lesson planning. At each presentation there were group discussions and additions to the problem presented.
06/07	End of the presentations and discussions about each proposed problem. The group, in a collaborative process and based on the reflections made about each proposed problem, chose two problems that, according to the consensus of the moment, best met the objectives of the lesson development regarding second degree equation for ninth grade students.
13/07	The chosen problems were discussed and reformulated when necessary. At this moment, the whole group tried to anticipate the students' reasoning and possible difficulties and tried to eliminate possible doubtful interpretations. The participation of the mathematics teacher, who supervises the school, was important to bring day-to-day situations and promote reflection with the future teachers.
Conducting the Class	
26/07 To 30/07	By decision of the PIBID/Foz group, the PIBID teacher was responsible for conducting the planned class with the ninth-grade students and the other members of the group were responsible for the observations. Remember that due to the COVID-19 Pandemic, the classes were watched by the pibidians remotely through the Classroom platform made available by the Paraná state government (Brazil).
Post Class Reflection	
03/08	On this day, we started the post-class reflection about the first problem developed and applied with the ninth-grade students. The future math teachers and the supervising teacher were able to discuss and reflect on the strengths and difficulties in carrying out the proposed lesson. It was also pointed out how the students reacted to the problem presented and how it was solved.
17/08	The second problem developed and applied to the ninth-grade students was discussed. After this reflection, each PIBID student was responsible for preparing a reflective report about the

	activities developed, highlighting the perceptions of the process, as well as what exceeded expectations and what was considered as a limitation in carrying out the activities developed.

Source: Organized by the authors, 2021

After the presentation of the summary of the activities and discussions that took place during the Lesson Study in PIBID, we will now present the analysis and discussion of the data in order to answer our research problem concerning the limits and potentialities of the Lesson Study in the Initial Training of Mathematics Teachers.

Data analysis and discussion

Based on the typology of teaching knowledge proposed by Shulman (1986, 1987), we developed our categories of analysis. Thus, in the following topics we present the evidence of the knowledge developed by the pibidians, through the participation-action in the PIBID, by means of the Lesson Study.

Knowledge of the teaching content (specific)

The academics, when proposing the lesson to be developed for teaching equations of the second degree, believed to know the subject well, however, when asked by the teachers (researchers) what is an equation, they could not answer. In addition, they did not understand the relationship between equation and function, not knowing how to differentiate them. In this regard, we chose a representative narrative from one of the participants:

> However, it was verified that there was no consolidation of the equation and function concepts, so the need to understand the historical development of function and equation was verified [...] With the intention of a real consolidation of the equation and function concepts there was an individual elaboration of an informal conceptualization of function and equation, therefore after several reflections, debates and researches together with the responsible teachers, the school's math teacher and PIBID students an informal definition of function and equation was elaborated. (A-7)[2]

Academic refers to the perception, by the PIBID group, of the need for a study using the History of Mathematics for the understanding and development of the concepts of function and equation. After this study and individual elaborations about these concepts, followed by

[2] To ensure the anonymity of the participants, the expression "A-number" is used to indicate the PIBID student.

socialization and reflections within/with the group, the following definition of function, equation, and equation of the second degree was collectively reached

> - A function is a formation law that establishes a relationship, which associates to each element of a numerical set A, called domain, a single element of a numerical set B, called codomain. The formation law is an algebraic expression formed by mathematical operations and that, by means of equality, relates two variables: independent (element of the domain) and dependent (element of the counter-domain). The Domain set provides the numbers for the substitution of the independent variable in the formation law, in which each value obtained in the substitution and in the development of this algebraic expression make up the Image set, this being a subset of the Contradomain.
> - Equation is an equality between mathematical expressions, containing at least one algebraic expression. The algebraic expression is defined by mathematical operations, numbers and one or more unknowns. Unknowns are any symbol used to represent an initially unknown value.
> - Second degree equation is an equality between mathematical expressions, containing at least one algebraic expression. The algebraic expression is defined by mathematical operations, numbers and one or more unknowns, the greatest exponent of which must be of degree two. Unknowns are any symbol used to represent an initially unknown value. In the second-degree equation up to two values satisfy the equality between the mathematical expressions.

As already observed by Vieira, Ponte and Mata-Pereira (2022) and Fernandez and Zilliox (2012), regarding the contribution of Lesson Study to the learning/deepening of the future teacher's mathematical knowledge, our students also signaled such learning. They would probably continue, if they did not participate in PIBID, and for some time, with a superficial and diffuse knowledge about the object of knowledge in question.

General pedagogical knowledge

The pedagogical knowledge cited by the students was many and diverse, both in the narratives and in the audio-recorded group discussions. Themes such as: a) the importance of 'arousing' the student's interest; b) the need to manage a class based on dialog and questioning by the teacher about the object of knowledge being discussed; c) stimulating student participation-action in the classroom; d) the organization and dynamics of the class held by the mathematics supervising teacher, a class developed by the PIBID group via Lesson Study; e) the importance of putting yourself in the student's shoes; f) the importance of contextualization to the teaching and learning process; g) about different means for teaching; among others, were identified. The following excerpts highlight this fact:

> The teacher tried to arouse the interest of the students in solving the proposed exercise, there was a very nice interaction of the students present in the classroom [...]. (A-3)
> The interaction occurred through dialogues and questions between students and teacher. (A-7)
> I was very surprised how much the math teacher stimulated the participation of the students during the lesson, asking questions, suggesting reflections, giving hints so that they could arrive at the answer. (A-8)
> The math teacher made the class extremely dynamic, where she constantly tried to ask the students questions to make them think about what was being worked on and participate in the class. (A-8)
> Many students, including those from home, participated in the class. The "difficulty" for the teacher was to pay attention to the students who were at home and at school at the same time, and to adjust the camera so that everyone could be seen. (A-9)
> [...] allowing the students to experience the classroom from the teacher's point of view, from the planning of the lesson to its execution, and this tends to lead to a more confident professional who is better prepared to face any adversity in the exercise of his or her function. (A-4)
> We can start by contextualizing the teaching, at the time of the application of the problems. (A-5)
> And as a future teacher, I could understand that there are many methods to teach this content, such as the one presented by Janice to the ninth-grade students (A-9).

Such knowledge, derived from research in Didactics, Psychology, Methodology, Sociology, etc., is important to future teachers, since the complexity of the teaching work requires, from the teacher, diverse knowledge beyond the specific, in our case, Mathematics. We emphasize that Lesson Study, by having the lesson as the main thread (its planning, realization and post-class analysis), ends up providing opportunities for problematization and reflection (NAKAMURA, 2019; PONTE; WAKE; QUARESMA, 2020) around this knowledge that is necessary for pedagogical practice, as well as for planning, since they are considered (or should be) in/during the preparation of the lesson.

Pedagogical content knowledge

Regarding pedagogical content knowledge, whose protagonism and elaboration is the teacher's (SHULMAN, 1986, 1987), the academics highlighted the following:

> I would use an object that could represent how we refer to the area or give an example of the school sports field. (The academic refers to the problem situation 2 "A football field has an area equal to 10800 m² and its length is 30 meters greater than its width. What are the dimensions (length and width) of this field?", prepared by the PIBID group and which constituted the lesson applied by the supervising mathematics teacher). (A-2)
> The first difficulty was in identifying the quantity of hot dogs (question 3) with the rereading of the problem situation and with the mediation of the teacher they identified as x, another difficulty was in establishing the relationship between the unit price of hot dogs and sandwiches (question 4), but through

> their attempts and the math teacher as "x" "the price of the sandwich is equal to the quantity of hot dog" they arrived at "the price of the sandwich is equal to the price of the hot dog". (The student refers to problem situation 1, "Peter bought 4 sandwiches at a certain price. He also bought a hot dog, which had the same unit price as the sandwich. The quantity of hot dogs bought was equal to the unit price of each sandwich. He paid two hundred reais bills and received R$8.00 in change. Let's analyze:", prepared by the PIBID group and which constituted the lesson applied by the supervising math teacher). (A-7)
>
> This experience highlighted that introductory activities more linked to the everyday life of the child and guided by questions or with more visual and illustrative demonstrations (such as geometric) are very important for the best development of second-degree equations for elementary school students, and to support this process as an academic and future teacher, I will put this knowledge into practice, both the exercises and the search for content in the history of mathematics. (A-2)
>
> [It was observed that problem solving allied to daily themes would be the best option to give meaning to the approached mathematical content, it was also taken into consideration the difficulty usually presented when solving a problem, in general, by the students, so it was decided to ask questions that directed the construction of the equation, where the answer to these questions would be the data collected in the problem. (A-5)
>
> We can also cite the importance of everyday topics and this style of solution as a way to apply to mathematical problems that may present themselves in the school future of these students, because they can create their own auxiliary questionnaires to collect the information of the proposed problem. (A-5)

It can be noticed, in the narratives, the transformation via formulation and presentation of the content by the supervising mathematics teacher at the time of addressing the problem situations in the classroom with her ninth-grade students. The PIBID students - who observed the supervising math teacher's practice during the class - identified these transformations, either in the use of visual resources to support the beginning of the student's understanding of problem situation 2, or through the teacher's mediation with continuous and different explanations to allow the student to identify the unknown quantity of problem situation 1. It was also highlighted the importance of the use, by the math supervising teacher, of themes from everyday life in order to relate school mathematics to routine situations, in an approximate and necessary attempt to the understanding of the students.

With respect to knowledge of the curriculum, no evidence of curriculum development by the students of the PIBID was identified. We believe this is due to the fact that they did not study/research curricular aspects when studying the object of knowledge addressed in the lesson developed via Lesson Study.

Knowledge of the students and their characteristics

During the meetings, discussions and reflections held by the PIBID group, the academics highlighted the importance of paying attention to the students, their characteristics and needs in the process of teaching and learning mathematics. Seeing oneself as a teacher, an important condition for Initial Training, was required from the students, when they elaborated the problem situations, they tried to anticipate the possible difficulties that ninth grade students would face when solving these problems, considering their characteristics, difficulties, etc. In one of the class observations made by the mathematics supervising teacher, the academics highlighted the following in relation to the difficulties of the ninth-grade students:

> The teacher had to ask questions several times for the students to assimilate the steps of solving the exercise. I observed a difficulty on the part of the students in manipulating the formula when out of order. (Refers to the Problem 1 situation). (A-10)
> The main difficulty was in relation to how to calculate the area of the rectangle, because in one of the classes, the students confused the formula for the area of the square with the formula for the area of the rectangle. (Refers to the Problem 2 situation). (A-6)
> One of the difficulties I observed was just with the question of developing the exercise with the perfect square trinomial method and in why the square root in the quadratic formula has to have a plus or minus in front. (Refers to Problem situation 1). (A-2)
> After the teacher explained the problem, I noticed that the students felt a little difficulty in organizing the necessary content to solve the problem. But this difficulty was dispersed when they were questioned with the elaborated questions besides the problem as an aid in solving it. (A-3)

During the group meetings, the PIBID students emphasized the importance of paying attention to these difficulties during the process of teaching and learning mathematics, which are inherent to the construction of mathematical concepts. Still on the characteristics of ninth grade students, many narratives highlighted the pandemic moment in which the PIBID was carried out, which resulted in different postures and characteristics of students who were present in person and remotely during the hybrid class developed by the math supervising teacher:

> The online students, through the Meet platform, also participated in the explanations with suggestions for answers to the questions developed on the board by the teacher, only to a lesser degree than those present in class. (A-5)
> The students participated a lot, but the students who were in the classroom, I noticed that the students who were at home watching the online class participated more when the students who were in class did not know how to answer some of the teacher's questions. At each question that was on the slide, the teacher asked the students and went back to the question so that they could identify the answer and, also, in the same question she asked several questions for the students to reflect and help them reach the

> expected answer. At each question from the teacher, the students interacted a lot by answering, but those who were in class with her were more active in participating, the students who studied from home seemed to me to participate much less. (A-8)

As Lesson Study is concerned with student learning, since its objective always aims at this end, it is essential to know the characteristics of the students, since these influence the process of mathematics teaching and learning. Still about the pandemic moment, the following topic will show how it impacted the lesson developed by the supervising math teacher.

Knowledge of educational contexts

Knowing the local reality and characteristics is necessary for the teacher's work, given the influence of these on daily life at school and in the classroom. As already mentioned, the PIBID, which is the training context in which the experience of Lesson Study was carried out, took place during the COVID-19 Pandemic. Thus, the students were unable to go to the field school and 'live' the school reality, as advocated by the PIBID. Even the observation of the class - which was conducted by the mathematics supervising teacher - by the students occurred remotely, via the Classroom platform. Thus, the narratives of the pibidians largely point to this pandemic context and its influence on the occurrence of the class:

> The main difficulty was the lack of infrastructure. Despite the experience of the math teacher, the technical limitations imposed on her sometimes-created obstacles. (A-4)
> In some moments, there were difficulties in communication and image transmission for the students who were online, because the Internet connection was bad. (A-6)
> If the problem situations had been applied to face-to-face teaching, the teaching and learning process would have been more satisfactory, because at times the Internet signal was bad, which completely compromised the transmission of the class. In fact, at one point, one of the students said that he couldn't understand the content because the transmission crashed a lot. In addition, most of the students kept the cameras turned off, which made the pedagogical investigation more difficult. (A-6)
> I noticed difficulties when doing multiplication 1.25x1.25=1.5625. Another difficulty is related to hybrid teaching: students in the classroom and students in the meeting, teacher Janice has to divide herself and usually cannot hear what the students in the meeting answer or ask. (A-7)
> It is remarkable the consequences of the pandemic in the classroom, less participative students in the hybrid modality, the difficulties that the teacher faces for sharing knowledge with her students. (A-1)
> Through this observation it was possible to notice that being a teacher is not an easy task, because in the school environment there is an encounter of several people, with different thoughts and ideas, and it is up to the teacher to make

> the articulation between students, social contexts, structural problems, knowledge among others in the classroom. (A-10)

The fact that the students pay attention to these contextual aspects - and this was largely made possible by the post-class analysis moment, which constitutes the third moment of the Lesson Study - ends up contributing to their education, as they begin to realize how complex the classroom is in which these contextual characteristics converge.

Knowledge of educational goals, purposes, and values

In relation to the last knowledge of the typology proposed by Shulman (1986, 1987), the knowledge of educational ends, purposes, and values, only in one narrative - presented below - we observed evidence of it:

> Thus, this experience was valid because during initial training it allows us to have a vision of the school organization, what its goals are, and how it can impact the lives of students, as well as providing reading about the teaching methodologies used by the current teacher, starting the reflective construction of our own teaching practice. (A-10)

We believe that the low, we would even say inexpressive, occurrence of this knowledge is due to the fact that the lesson study carried out with the PIBID group was more concerned with the planning, execution and post-class analysis than with the specific historical-philosophical aspects and those articulated with the purposes, objectives and educational values.

So far, we have presented the potentialities - represented by this range of pedagogical knowledge indicated by the academics - of the theoretical-practical aspects of Lesson Study in the initial training of mathematics teachers.

Regarding the limitations of Lesson Study, given our context and the historical moment of its emergence, we believe that the discussions and reflections would have been enhanced if they had taken place in person. On several occasions, when the weekly meetings were held through the Microsoft Teams platform, the scholars had difficulty accessing and/or staying on the platform due to Internet connection problems. In addition to this connectivity issue, we also noticed that the students were sometimes dispersed, as maintaining attention on a computer screen is not an easy task, especially considering that many of them did not have adequate space in their homes to attend the meetings.

Another limitation was the difficulty in observing the solution of problem situations (1 and 2) by the ninth-grade students of the supervising math teacher. As mentioned above, the Pibidians observed the interaction between the teacher and her students from a distance, and some of these students were also present from a distance. Thus, the observation of the students' solution of the mathematical tasks, an important moment in the lesson study, suffered considerable impact. However, even considering this difficulty, the data presented above indicate, within what was possible, the occurrence of this observation followed by reflection by the students of the PIBID.

Final considerations

The constitution of the teacher is a complex work that requires a long period of formation and that continues throughout the professional career. Initial training, considered as the first formal contact aimed at preparing teachers, must contribute to this constitution. In this perspective, the PIBID aims to contribute to this formation by integrating the university and the school of basic education, as well as its different actors (teachers and students of basic education, students and university teachers).

We believe that the implementation of the Lesson Study, during the PIBID, has contributed to the initial formation of our students. The diverse knowledge of teaching that they have learned, as well as the possibility of thinking (and acting) as a teacher when planning a lesson, in its various details and conditions (the complexity of the proposed mathematical activity, the anticipation of the student's thought-resolution, the organization of the didactic actions to be carried out, the time of the lesson, the lesson evaluation, etc.), have contributed to their education.

Thus, what we observed - when we carried out this formative experience during the PIBID, through Lesson Study - is in line with what Nascimento, Carvalho and Ramos (2022) point out when they indicate that Lesson Study is a collaborative and reflective formative context, important for the initial formation of mathematics teachers.

Finally, it is worth noting that the PIBID is an important project for the valorization of undergraduate courses and teaching, but this "model" of training is experienced only by some students of the undergraduate course. We understand that this was also a limiting factor of the proposal based on Lesson Study, since we did not reach all the students of the undergraduate mathematics course. Considering this fact and the importance of the process experienced by the

students of PIBID, it is necessary to think of alternatives so that the Lesson Study can be incorporated as a proposal of the Undergraduate Course and/or as a proposal of a subject, such as the Supervised Internship subject.

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