From *Lesson Studies* to an *Italian Lesson Study*: a cultural transposition process

Arzarello, Ferdinando
ferdinando.arzarello@unito.it
https://orcid.org/0000-0002-0913-7500
Department of Mathematics "G. Peano"; University of Torino; Torino, Italy
Bartolini Bussi, Maria Giuseppina
mariagiuseppina.bartolini@unimore.it
https://orcid.org/0000-0002-3257-1348
Department of Educational and Human Science; University of Modena e Reggio Emilia; Modena, Italy
Funghi, Silvia
silvia.funghi@unito.it
https://orcid.org/0000-0001-8549-8474
Department of Mathematics "G. Peano"; University of Torino; Torino, Italy
Manolino, Carola
c.manolino@univda.it
https://orcid.org/0000-0002-8711-4638
Department of Social and Human Science; University of Valle d'Aosta; Aosta, Italy
Minisola, Riccardo
riccardo.minisola@unito.it
https://orcid.org/0000-0002-1105-4843
Department of Mathematics "G. Peano"; University of Torino; Torino, Italy
Ramploud, Alessandro
alessandro.ramploud@dm.unipi.it
https://orcid.org/0000-0002-9161-4213
Department of Mathematics; University of Pisa; Pisa, Italy

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From *Lesson Studies* to an *Italian Lesson Study*: a cultural transposition process

Abstract

In this article we trace a path that reports the work of Transposition of the Lesson Study (LS) from Eastern to Italian culture through the perspective given by the approach of Cultural Transposition. In it, the encounter with a culture different from one's own is seen as a means capable of raising new and deeper reflections by teachers on their own educational habits and practices. We call 'Italian Lesson Study' (ILS) the product of this process: it is not established a priori, but depends on the requests to participants that the researchers, as teacher educators, have chosen to graft into it. In particular, we present 4 examples of ILS realized in different school grades (1,5,8,9), to highlight how each LS team responded to these requests through their design choices concerning the research Lesson and the critical reflections of the different participants. Starting from this description, in the conclusions we outline some common features and differences among the examples in order to highlight more general reflections about the possible ways of implementing LS in cultures different from the Eastern ones.
Keywords: Lesson Study. Cultural Transposition. Italy. China and Japon. Mathematics Education.

Del *Lesson Studies* al *Lesson Study italiano*: un Proceso de Transposición Cultural

Resumen

En este artículo trazamos un recorrido que da cuenta del trabajo de Transposición del Lesson Study (LS) de la cultura oriental a la italiana a través de la perspectiva dada por el enfoque de la Transposición Cultural. En ella, el encuentro con una cultura diferente es visto como un medio capaz de suscitar en los docentes nuevas y más profundas reflexiones sobre sus propios hábitos y prácticas educativas. Llamamos al producto de este proceso el "Lesson Study Italiana" (ILS): no está establecido a priori, sino que depende de las solicitudes a los participantes que los investigadores, como formadores de docentes, eligieron implantar en él. En particular, presentamos 4 ejemplos de ILS llevados a cabo en diferentes años escolares (1, 5, 8, 9), para resaltar cómo cada equipo de LS respondió a estas solicitudes a través de sus elecciones de diseño sobre la Clase de Investigación y las reflexiones críticas de los diferentes participantes. En base a esta descripción, en las conclusiones esbozamos algunas características comunes y diferencias entre los ejemplos, con el fin de resaltar reflexiones más generales sobre las posibles formas de implementar LS en culturas distintas de las orientales.
Palabras clave: Lesson Study. Transposición cultural. Italia. China y Japón. Educación matemática.

Dos Lesson Studies ao Italian Lesson Study: um processo de transposição cultural

Resumo

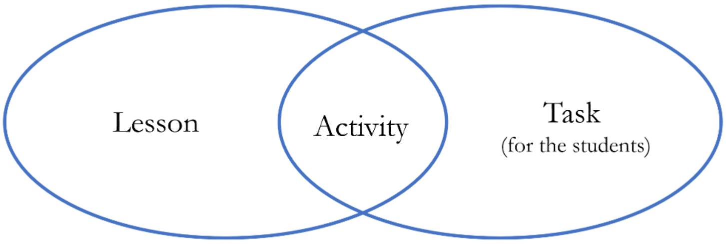
Neste artigo traçamos um percurso que relata o trabalho de Transposição do Lesson Study (LS) da cultura oriental para a italiana através da perspectiva dada pela abordagem da Transposição Cultural. Nela, o encontro com uma cultura diferente é visto como um meio capaz de suscitar novas e mais profundas reflexões dos professores sobre seus próprios hábitos e práticas educativas. Chamamos de "Italian Lesson Study" (ILS) o produto desse processo: não é estabelecido *a priori*, mas depende das solicitações aos participantes que os pesquisadores, como formadores de professores, escolheram implantar nele. Em particular, apresentamos 4 exemplos de ILS realizados em diferentes anos escolares (1, 5, 8, 9), para destacar como cada equipe de LS respondeu a esses pedidos por meio de suas escolhas de design sobre a Aula de pesquisa e as reflexões críticas dos diferentes participantes. A partir dessa descrição, nas conclusões delineamos algumas características comuns e diferenças entre os exemplos, de forma a destacar reflexões mais gerais sobre as possíveis formas de implementação do LS em culturas diferentes das orientais.
Palavras-chave: Lesson Study. Transposição cultural. Itália. China e Japão. Educação matemática.

The problem of "cultural implicit factors" in the transposition of the Lesson Study

Japanese and Chinese Lesson Study are fundamentally rooted in their culture of origin. In these contexts, the school has very specific structure and habits, often quite different from those of Western school systems. Specifically, as far as Italy is concerned, the main group of its specificities, compared with other countries' ones, particularly with those in Eastern countries, concerns the school organisation, as listed below:

1. The Italian Indicazioni Nazionali (National Guidelines – MIUR, 2012) indicate only long-term objectives (Minisola & Manolino, 2022), hence Italian teachers are not used to plan a lesson of limited time in detail.
2. The structure of the school day varies with the order of the school: a structure segmented into 'lessons' of 45-60 minutes dedicated to a single subject, different from the previous and the following lesson, is used in secondary schools, where each discipline or group of similar disciplines (e.g., Art; Mathematics and Physics; Music; History and Philosophy, etc.) is taught by a different teacher , but does not exist in primary school, where generally there is only one generalist teacher or at most two teachers per class. From an institutional point of view, the term 'lesson hour' is often accompanied by the locution 'hourly unit', 'hour', or 'minimum hourly units'. On the other hand, in the daily life of teachers, this term can have a vaguer meaning, some of which overlaps with that of 'activity' (fig. 1).

Figure 1 - Partially overlapping meanings in Italian school terminology



Source: Elaboration by the authors

3. Freedom of teaching is enshrined in the Italian Constitution: each teacher can choose teaching methods and the order of contents to be presented; as well textbooks are chosen by the teacher according to those that (s)he thinks the best for her/his methods.
4. Inclusiveness in school is guaranteed by law: students with difficulties attend only regular schools and lessons, possibly with the help of a dedicated teacher (special schools have been abolished since many decades).
5. Teachers' background varies: at primary school teachers have a pedagogical background; at secondary school mathematics is taught by teachers who have a master in a STEM discipline.
6. Teachers work mainly alone.
7. Teaching design is lax when not absent.

8. Teachers' professional development courses are often theoretical and not centred on everyday teaching practices. Although it is sometimes delivered in the form of workshops, teacher education ultimately leaves it up to the teacher to work out how and when to implement teaching proposals in the classroom. Detailed and context-specific planning is always the responsibility of the individual teacher alone.

These features have deeply puzzled the Italian researchers[1] in mathematics education (Mellone et al., 2021) who in their scientific activities have become in contact with the Lesson Study (herein after referred as LS). They seemed to be possible stumbling blocks for a suitable transposition of LS into the Italian schools. In particular, a reflection about this transposition was initiated by the first group of researchers who introduced LS in Italy since 2012, namely the team of M.G. Bartolini Bussi from Reggio Emilia (Bartolini Bussi et al., 2017; Ramploud et al., 2022), to which two other authors of this article belong from the beginning of their Italian LS project. Later, their experience inspired another team of researchers in Turin, three of whom are the other authors of this article (Minisola, 2016; Manolino, 2021). A joint experience so started (Arzarello et al., 2022), which is still continuing and essentially represents the core part of the LS activities in Italy, to which other small groups joined. This paper sketches the main ideas that motivated these experiences featuring, so to say, the Italian LS (ILS henceforth), and illustrates them through some concrete examples of LS experiences in the different school grades, coached by the authors in these last years.

Our approach to LS began participating as observers to public LS realised in Far East countries; starting from our observations, we successively tried to imagine how such a practice could be recontextualized within the Italian school context (see Bartolini Bussi, Sun & Ramploud, 2014; Mellone & Ramploud, 2015; Mellone, Ramploud & Carotenuto, 2021). Our encounter with the "foreign" teaching practice triggered a twofold process.

From the one side, we "recognised" in some components of Eastern LS aspects that were coherent with our experience and to our values as teacher educators: e.g., the need to

[1] In this paper we distinguish between researchers in mathematics education, teacher educators and facilitators: the latter are those responsible for coordinating the work of the LA team; of course, a facilitator can be a researcher. We use also the term *didactician* to indicate the researcher as engaged in a professional development course for teachers, (following Jaworski & Potari, 2021). We prefer this term since in Italy, educator [*educatore*] is a professional figure that is different from the teacher, because his/her role is aimed to reduce the gap between students with special needs and the others; an educator [*educatore*] is not a discipline-expert but an expert in taking care of the human relationship with students (see Ramploud et al., 2021). In the following we will use the Italian word when we refer to this specific professional figure, in order to distinguish it from researchers in mathematics education, teacher educators and facilitators.

elicit teachers' reflective approach to teaching practice; as well what we call 'didattica laboratoriale' (laboratory teaching) practice. This practice consists of an elaboration of the notion of laboratory, which goes back to Emma Castelnuovo (2008) and was further developed in a professional development program for teachers, developed from 2001 to 2012 with the support of the Italian Ministry of Education and of the professional association of Italian Mathematicians (UMI). According to it (Arzarello et al., 2012), the mathematics laboratory is not necessarily a physical place different from the classroom; instead, it is seen as a structured set of activities for the construction of meanings of mathematical objects. According to this view, the mathematics laboratory environment is in some way comparable to that of a Renaissance workshop, where apprentices learned by doing and seeing, communicating with each other and with teachers. With an evocative metaphor we can say that the classroom becomes like a polyphony performance in a concert hall (Bartolini Bussi, 1996), where the students are the musicians, the teacher is the conductor and the materials that are used are the musical instruments. The construction of meanings cannot reside solely in the instrument, nor can it emerge solely from the interaction between student and instrument. Meaning lies in the purposes for which the tool is used, in the plans that are made for using the tool; moreover, the appropriation of meaning also requires individual reflection on the objects of study and the proposed activities.

Another feature of the 'didattica laboratoriale', which we found coherent with the Eastern LS, was the fact that in its development teachers had to keep a logbook, where they reported with some detail the design of their activities in the classroom, what had happened during their concrete development with students, and their comments about the problems they had encountered in it. Of course, the logbook is not the same as the documentation in the LS but surely has some affinity with it, so that – we as researchers thought – it could be an aspect of the 'didattica laboratoriale' which could improve an easier transposition of the LS into the Italian schools.

From the other side, reasoning by contrast about Eastern LS, we also recognized conflicting aspects with respect to our school culture: e.g., focus on short-term goals, detailed lesson planning, teamwork, teacher education through participation in real lessons. A small survey we developed in 2019 with some teachers confirmed that these can be real discordant features in Italian teachers. The data from this experiment highlighted the aforementioned issue on the meanings of the terms "lesson" and "activity", suggesting an overlap in the meaning of the two terms (see fig. 1). After this, we organized two surveys with different teachers: one with teachers participating in a professional development project of the

Department of Mathematics at the University of Turin (44 responses), and another with mathematics teachers at a technical institute and scientific high school in Turin (27 responses). The answers confirmed that, even for in-service secondary school teachers, the meaning of the term "lesson" is not shared with researchers: the majority of the teachers in fact "quite agree" or "very much agree" with the statement "a lesson can occupy several hours spread over several days" (64%) and "completely disagree" or "quite disagree" with the statement "a lesson is something that is articulated in the hours I have in a specific class on a specific day" (70%). This cultural fact is something to be considered when transposing Lesson Study in the Italian context (and which may prove problematic in other contexts as well). Especially as, in the Japanese and Chinese Lesson Study, the word "lesson" assumes a very specific connotation, being a well-defined didactic research object, and which is therefore well "circumscribed" in space and time (the 'research lesson'[2] takes place in a very specific class and time). In the Italian context the "lesson" has no well-defined time boundaries and may last much more than one hour. These conflicting aspects of Eastern LS with respect to our teachers' habits, from our viewpoint as teacher educators constituted excellent "triggers" for teacher education from a perspective of "empowerment" of teachers' critical thinking. Indeed, they allowed us to discuss with teachers' issues concerning the usefulness of a detailed lesson planning, the possible advantages in scaffolding long-term goals into short-term ones, or in increasing the occasions for a collaborative lesson design in their activities. This "empowerment" was important to us as teacher educators in our context, because of teachers' freedom of teaching established by the Italian Constitution. Some of these aspects had also emerged as needs in previous teachers' professional learning courses: the teachers themselves, emphasizing the purely theoretical nature of these courses, complained about a lack of reflection on daily teaching practices (as also outlined in the 2018 OECD Teaching and Learning International Survey – OECD, 2020). LS somehow seemed to provide an overall context for working on all these aspects simultaneously. This made LS an extremely interesting teacher education practice for us. However, already from the first experiments, it became evident for us the need to consider LS's cultural situatedness to redefine its purposes and to discuss a suitable "operating space" to act within the Italian cultural context.

[2] From here on, we refer to the lesson co-planned, implemented and discussed by the LS teams within the Lesson Study context as the "research lesson".

Indeed, reflecting on the potential of LS observed in Far East countries, our idea was not that of comparing different cultures "translating" teaching practices, textbooks, etc. from one culture to another, but that of attempting to highlight differences. In fact, these differences should be those bringing out the possible *unthought* of our own culture – we have to be aware that this process concerns *our* culture, our way of living, our perspectives on the world – prompting us to look at ourselves in a new way. In fact, our work is inspired by the reflection of F. Jullien (2006) who writes that "it is not a matter of comparative philosophy, of putting different conceptions in parallel, but of a philosophical dialogue where each thought, in meeting the other, questions its own unthought" (p. 8). Therefore, we gave a particular attention to the peculiar features of Italian school listed above (Bartolini Bussi & Martignone, 2013) to find a suitable way of implementing the LS. This process is called *Cultural Transposition* (CT), which provides a perspective whereby it is possible to "import" foreign teaching practices, using them consistently with the new context, but retaining some of their conflictual aspects with the new cultural context (Ramploud et al., 2022). CT means not to "perform a comparative study or a slavish import-export of mathematics education methodologies and tools between different countries, but rather to open a dialogue between two different cultures in which every thought, meeting the other culture, questions its own unthought" (Mellone et al., 2021, p. 382). Its aim is to exploit the cultural conflictual aspects to trigger teachers' questioning about their own teaching practices and educational intentionality (for more details see Mellone et al., 2019). We emphasize that in a CT process the choice of conflictual aspects to be retained is not given *a-priori*. Those described in this paper are our choices, based on our goals as teacher educators. Others may make different choices and obtain different "products". In our case, CT process was related to specific choices regarding the structure of the Lesson Plan to be used, the length of the lesson to be planned, and specific requests to be fulfilled by participating teachers during the various phases of LS: the "product" of this CT process at the Italian level was the ILS. For the aim of this paper, and in order to provide the reader with the interpretative keys to better grasp the necessary cultural transposition from LS to ILS (for more on LS Cultural Transposition, see Ramploud et al., 2022), in Table 1 we present two columns: "*Requests*" (left column) and "*Rationale*" (right column). With the expression "*Requests*" we mean those elements that, as didacticians, we identified as necessary to construct LS coherently with the Italian didactic culture; therefore, we requested the teachers participating to ILS experiments to pay specific attention to these elements (this is why we chose to call them "requests"). With the expression "*Rationale*" we indicate the motivations and intentionalities that guided the identification of each "Request".

Table 1 - Requests characterising ILS and related rationale

Requests	*Rationale*
[1] Request for lesson structured with a part dedicated to student's hands-on activities to solve a given task, followed by class discussion [2] Request for particular attention to the choice of tasks and materials provided to students.	Italian tradition of research on the 'didattica laboratoriale', where there is a specific attention to the use of instruments and to the mathematical discussion (problem solving is coupled with a collective discussion on the mathematical content).
[3] Request for particular attention to the choice of lesson short-term objectives to be achieved within a single lesson.	The Italian Indicazioni Nazionali indicate only long-term objectives; Italian teachers are not used to plan a limited time lesson in detail.
[4] Request for teachers' explicit reflection on educational intentionality underpinning their choices for lesson design.	Freedom of teaching enshrined in Italian constitution: each teacher can choose teaching methods and the order of contents to be presented, so that it was useful for us as teacher educators to make those choices explicit to teachers.
[5] Request for the analysis of the classroom context.	Italian school inclusiveness[3] and Italian teachers' freedom of teaching.
[6] Request for structuring the observation process.	Research in the field of pedagogy in Italian context highlighting the need for teachers to learn how to conduct a systematic observation (Braga & Tosi, 1998; Cardarello, 2016).

Source: Elaboration by the authors

Starting from these requests we describe four paradigmatic ILS examples: in order to illustrate how each LS team can choose to answer to these requests, in the description of each example we indicate, for design choices and for critical reflections raised by the participants, the request they refer to. These four examples are singular experiences of LS, without referring to cycles of LS's, except the small reference in the fourth example. Cycles are present in our experiences, but to analyse them we have used a wider frame, whose illustration is beyond the aim of this paper. The interested reader can see Arzarello et al. (2022) for this analysis. We hope that these examples can inspire the Brazilian researchers and teachers to elaborate their own experiences with LS, not translating them passively into

[3] Italian school is "inclusive", in the perspective that there are not special schools for students with disabilities or special needs. Therefore, at every school level, the same class can include children with learning difficulties, with physical and mental disabilities, children that are not Italian speakers, and so on.

their schools but using them to ‘bring out the unthought of their own cultures’, as it has been the case for us.

First example: LS on difference (1st grade)

Context: This LS was developed in a 1st grade class (second half of the school year). It was composed of 18 students, one of whom with visual impairment.

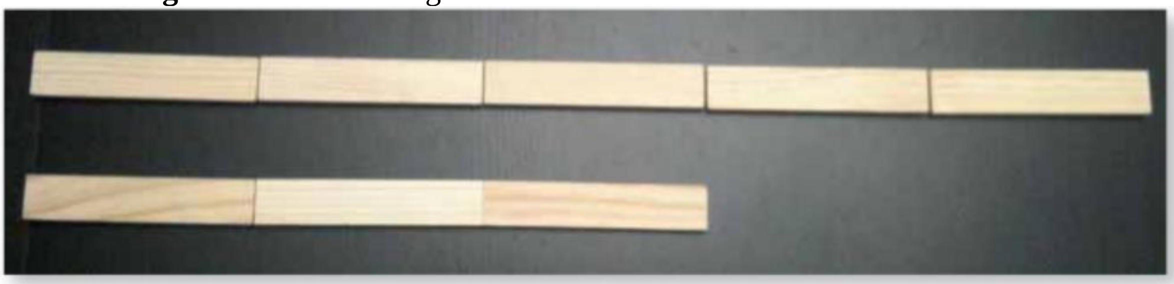
LS team: The team was composed by 6 primary school teachers, 2 *educatori*, and 2 didacticians. Among them, 5 teachers were observers, 1 *educatore* filmed, the other *educatore* co-conducted the research lesson with the pilot teacher (by ‘pilot teacher’ we mean the teacher who conducts the research lesson); the *educatore* had already known the class in previous lessons.

A priori analysis

Aim: The aim of the research lesson was introducing students to difference. The pilot teacher was interested in providing students the opportunity to face and solve a problematic situation not only by subtraction but also by strategies exploiting addition (e.g., counting on strategies) or other possibilities (request [4] in Table 1). Teacher, in particular, knew ongoing studies on Chinese didactic practices, as one problem multiple solution tasks (Bartolini Bussi et al., 2014; Venkat et al., 2018).

Main idea: The research lesson design was developed around the core idea of providing to each child a set of KAPLA®s greater than 10 and asking them how they could “make equal” two rows of KAPLA®s[4] like in fig. 2 (request [2]).

Figure 2 - Initial configuration of KAPLA® rows shown to the class



Source: Picture taken by the authors

[4] It is a set of wood rectangles having all the same dimensions (see: https://www.kaplaplanks.co.uk/the-kapla-game_i13.html). These wood rectangles can be coloured or not. In our case, see Figure 2, we used the non-coloured set.

The LS team came to choose this particular task through a discussion highlighting the complexity of a-priori task analysis for teachers (requests [2] and [3]). The participants discussed a lot before finding a "good" question , since in the beginning all the proposed task formulations seemed not really opening to multiple solution, but indicating instead a sort of "preferred" strategy (for instance, teachers argued that the question "How many KAPLA®s of difference there are between the two rows?" could suggest mainly subtractive strategies rather than additive strategies), creating a sort of "hierarchy" between "right" strategies and "wrong" ones. Eventually, a participant formulated the task in this way: *«In my opinion, the way to make them give more solutions is asking "how would you make these two rows equal?". In this way we could get as answers either taking away 3 from the upper row, or adding 3 to the lower one... »*. This formulation was recognized by all the team as a real multiple solution problem, so it was chosen as the definite task formulation. It is noteworthy that this discussion allowed this teacher to better analyse the possible task formulations in the light of the intentionality at stake, emphasizing the importance of the *a-priori* analysis to elicit teachers' reflection on their own intentionality (request [4]). Moreover, this choice was aligned with the pilot teacher's aim to propose a task involving both additive and subtractive solution strategies (requests [2] and [4]). The team expected for the "difference" between the two rows of KAPLA®s to emerge through (at least) three possible solutions:

1. Removing a certain amount of KAPLA®s (at least, 2 from the longer row);
2. Adding a certain amount of KAPLA®s (at least, 2 to the shorter row);
3. Distributing a certain amount KAPLA®s between the two rows (at a minimum, move 1 KAPLA® from the longer row and place it in the shorter row).

After this manipulation with KAPLA®s, the team chose to ask the same question but using a representation of rows of rectangles drawn on paper (see fig. 3). The team argued that the introduction of this different material (paper) could allow students to use different solving strategies with respect to the previous manipulation, i.e. cutting the paper or drawing on it. This change of materials was aimed at highlighting the possibility of multiple solutions among different students, and the possibility for each student to make different choices in relation to different materials (request [2]).

Figure 3 - Representation on paper given to students in the second part of the activity

Source: Elaboration by the LS team

This manipulation with materials was planned to be organized as a small-group work, even if each child was given his own material. At the end of this part, the team planned to implement a whole-class discussion about different groups' strategies (request [1]), focused especially on students' argumentation in the two cases. The conclusion of the research lesson was planned to be dedicated to a summary by the pilot teacher of the different solving strategies, aimed at pointing out the possibility of having different alternatives for solving a single problem (requests [4] and [3]), but above all at introducing a first possible formalization with symbolic language, such as: 5-2=3 or 5-4=3-2 for subtractive strategies; 3+2=5 or 5+1=3+3 for additive ones; 5-1=3+1 for distributive ones.

Description of the ILS

To collect data about the research lesson development, the team decided to film the lesson placing a camera in a classroom corner, and collecting the 5 observer teachers' field notes (request [6]). The team chose to begin the lesson projecting the photo reported in fig. 2 on the multimedia whiteboard of the classroom, dividing students into 2 groups of 5 and 2 groups of 4, and giving to each child 12 KAPLA®s. To facilitate the work for the visually impaired student, the team chose to cover his desk with a black sheet of paper, to make it easier for him to see the light-coloured KAPLA®s (request [5]). First, the students were asked to reproduce the projected configuration with KAPLA®s; then, the pilot teacher posed the problem of how to make the two rows of KAPLA®s equal (request [2], task). The *educatore*, for this part of the lesson, was going around the class helping students and making questions aimed at supporting students' reasoning on the task. As anticipated by the team in *a priori* lesson analysis, the task allowed for multiple solutions to emerge within the same group. For instance, of two girls that worked on the same desk, one added 2 KAPLA®s to the shorter row, obtaining two rows of 5 KAPLA®s each, while the other one removed a KAPLA® from the longer row and moved it to the shorter one, obtaining two rows of 4 KAPLA®s each. Then, the pilot teacher proposed to change the material, namely, to work on the representation reported in fig. 3. Each student was given a sheet with this representation. Introducing a different material, students came often up with different solution strategies with respect to what they did with KAPLA®s, even though the two tasks were very similar. Data collected by observers on the work on paper of the same girls mentioned previously revealed that neither of them repeated the strategy used with KAPLA®s: both, in this case, chose to cut the 2 rectangles in the longer row in order to make two rows of 3 rectangles each.

At the end of this small group activity, the teacher dedicated some time to the discussion of solving strategies (request [1]), paying particular attention to students' argumentations. Then the teacher summarised students' different solving strategies, pointing out the possibility of having different alternatives for a single problem (request [4]). This recapitulation of different solutions showed the possibility of exposing students to a situation where the individuation of the "difference" could be related not only to a subtraction process, but also to strategies such as counting on or moving an element from the longer row to the shorter one. Moreover, this part of the lesson was dedicated to introducing an initial formalisation of subtractive, additive or "distributive" solving strategies, through the operations of addition and subtraction: e.g., 5-2=3 (subtractive strategy); 3+5=2 (additive strategy); 5-1=3+1 (distributive strategy).

Once the video recordings were shared and watched by participants, they met to discuss and analyse the research lesson. The aim was individuating the strengths and the critical points of the design and identifying crucial points for the future development of class activities. In the discussion, the following crucial reflections emerged:

1. *Issues of time and of rigid lesson schedule*. Some participants pointed out that in Chinese LS (for example the one described in ICMI study 23), students' work on the proposed tasks is really fast with respect to Italian students' times. The observers suggested to consider reducing the number of proposed tasks in one lesson, to obtain a more profitable time organization (request [3]). These reflections led the team to emphasise the strategic importance of *a priori* analysis, retained very important to adequately implement the lesson as planned (request [3]).
2. *Importance of* a-priori *lesson analysis to choose materials* (request [2]). The participants realised to have paid so much attention to the analysis of KAPLA®, but to have underestimated the importance of other tools – for instance, the role of the multimedia whiteboard, which was fundamental for tracking recapitulations and argumentations, was considered at first only as part of the classroom setting (request [2]). However, the shift from using KAPLA® to using rectangles representation on

paper was retained effective in making each student experience different solution strategies to similar tasks (request [4]).

3. *Students' group work*. This aspect proved to be directly related to the analysis of materials. In fact, giving to each student their own individual set of KAPLA® and their own rectangle representation did not support group work: most students solved the problem individually, and students' interaction was mainly aimed at borrowing ideas, not at a real collaboration in the solving process (request [1]).

Concerning the importance of time, the team highlighted that also the interaction between the adults involved in the experimentation was conditioned by strict timings and by the (perceived) need to accomplish the initial plan. In particular, the pilot teacher emphasised that, in her opinion, the crucial part of the research lesson was reached in the end, when children's attention was already waning (request [1], overall design of the research lesson structure). This ILS, nevertheless, seemed to the participants a satisfying first attempt, because there were not many downtimes, and the proposed tasks attracted the attention and interest of the entire class. From this perspective, the participation in the activity of all children, even those with particular difficulties, showed that ILS is compatible with an inclusive school (request [5]), as long as the designed task sequence can be adjusted according to students' different times. The most difficult point seemed to be the discussion part, where the teacher can have difficulties in respecting planned timings, because it was difficult for the team to anticipate students' answers and the length or the richness of their argumentations. Therefore, at the end of the discussion meeting, the team identified two possible CT of LS to be chosen in a *priori* analysis: 1) one leaving more space to the discussion with students, aimed at raising important reflections and observations to be further developed in the following activities, less focused on formalization; 2) one focusing more on the core mathematical knowledge formalization. In this second case, the task should be chosen carefully to allow a less extended class discussion and more time for formalization (request [3]).

Second example: LS on calculators (5th grade)

Context: This LS was developed in a 5th grade class of 16 students, some with disabilities or learning disorders, of a school following the "No Schoolbag" model (see Schiedi, 2021). Students were used to work in small groups and in pairs (it is a typical feature of the "No

Schoolbag" model: the class setting is with tables for 4/5 people, not individual desks). The LS was carried out in November 2021.

LS team: The team consisted of 5 primary school teachers, 1 didactician and 1 master's student in mathematics in the role of facilitator. Three out of the five teachers were working in grade-5 classes that year. A support teacher of one of the children with mathematical learning difficulties was also present in the classroom during the research lesson (request [5]), albeit not being part of the team.

A priori analysis

Aim: The research lesson stemmed from the teachers' reflection about some educational goals to be reached at the end of 5th grade, as indicated into the Italian *Indicazioni Nazionali* (MIUR, 2012), emphasizing that "The conscious and motivated use of calculators and computers should be encouraged appropriately from the earliest years of primary school, for instance to check the correctness of mental and written calculations and to explore the world of numbers" (p. 60, our translation). The aim of the research lesson was to overcome the prejudice, widely spread among both students and parents, that the calculator should not be used in mathematics education because it facilitates calculations too much. The research lesson was intended to make students realise that the use of calculator not only allows to find the results of operations, as they commonly believe, but that it can enable them to reflect on their way of calculating, facilitating metacognitive reflection.

Main idea: The belief about the detrimental influence of the use of calculators on students' mathematics learning is not completely without foundation, but the team wanted to provide an example of lesson where the use of calculators cannot exempt students from activating their reasoning to solve the task at stake. In particular, the team thought that asking students to perform, for instance, a division without the use of the division button (i.e., ":") could completely change students' beliefs about calculation. Indeed, in this case, in order to find the result of the division, students needed to activate competences related to the different meanings of division, knowledge of the properties of operations and knowledge of our positional and decimal number system.

Description of the ILS

To collect useful data to analyse and discuss the development of the research lesson, the team decided that the researcher would have filmed the lesson with the tablet camera while observing one of the groups. The other groups of students would have been observed by the other team members, taking notes on the spot (request [6]).

In the teachers' view, a work on the use of calculator could not be reduced to just a one-hour lesson, hence the Lesson Study was part of a broader teaching programme (request [3]). The research lesson was preceded by some activities aimed at making students familiarise with the use of the calculator and the main functions of the buttons. In the research lesson the focus was on two activities inspired by "Matematica 2001" (a compendium of innovative teaching activities and assessment proposals edited by the UMI – Arzarello et al., 2012). The following are the proposed activities with related tasks and educational intentions (request [4]).

Figure 4 - The two tasks of the "Type and see" activity

Type	See
"On" button	
1	
2	
5	
X	
8	
3	
=	

Operation: ………………………………

Type	See
	0
	8
	86
	86
	2
	24
	62

Operation: ……………………………

Source: Elaboration by the LS team

The first activity was called "Type and see". The 'type/see' pattern was proposed as a tool to understand the difference between what the person does and what the calculator does. This activity was aimed at making students aware of the difference between what the calculator display shows and the way they would hand-write their calculations on paper (request [2]).

On the basis of an initial illustrative example performed by the teacher, showcasing the use of the 'type/see' pattern for computing 17+23, the team proposed the first activity by providing each child with a worksheet with the two tables in fig. 4 and the request to fill in the table step by step and to write down each performed operation. The planned time was 10 minutes. The team chose to make all the students perform the same example at first in order to make every student understand the meaning of the task of "filling in the table" (request [5]). The purpose of the task was in fact to make students reflect on the calculator's actions. The choice to have all the students writing for each step of the calculation was made to avoid dips in attention in the transition from the example to the subsequent problem solving. The LS team chose 17+23 because they considered it simple and understandable as an example: they considered the sum to be the "easiest" operation, and the numbers chosen were considered easy to be summed up without performing written algorithms (request [2], task). In this way students could check whether what they expect by mental calculation corresponded to what they got on the calculator, and thus they could pay their attention to the operation of the calculator. In addition, the numbers were chosen to involve 4 different digits in order to avoid confusion between them (request [2], task). Concerning the first table to be autonomously filled in by students (on the left in fig. 4), it was intended to propose a multiplication not easy to perform with mental calculations (request [2], task). In the second table (on the right in fig. 4), the "see" column was proposed already filled in and the students had to fill in the "type" one. In this way, the team aimed to test whether the students were able to go backwards, following the correlation between sequence of typed buttons and performed operation (request [3]).

The second activity was called "The mischievous calculator". It required performing operations, but for each task the use of the button for the operation at stake was hindered: in this way students must find alternative strategies to compute the considered operation (request [2], task). The 3 proposed tasks were:

1. 47*3 (use of "*" button hindered). With this task, the team expected students to implement different strategies: using repeated additions, eventually with the commutative property of the addition and some factor decompositions.
2. 47+7 (use of "7" button hindered). With this task, the teachers meant to explore the students' competence in applying the properties of addition. They purposely chose to include the digit 7 in both addends to see whether the children used the same strategy for both numbers or they reasoned differently about the two numbers (request [3]).
3. 250:5 (use of ":" button hindered). The teachers were aware that this prompt was considerably more difficult than the previous two. The team chose not to use division giving non-zero remainder, because they had agreed that this prompt would be an opportunity for further exploration with the calculator (request [3]). The choice of numbers was designed to ensure an easy mental computation.

Students then were asked to write down their reasoning, to choose one spokesperson per group to explain it to the rest of the class, and to justify the solution strategy (request

[1]). Figures 5, 6 and 7 show different solutions provided by students' groups to solve the second activity.

Figure 5 - First task solutions.

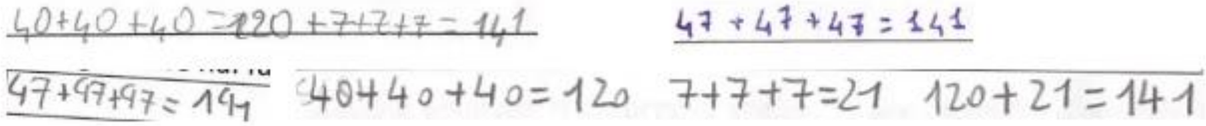


Figure 6 - Second task solution

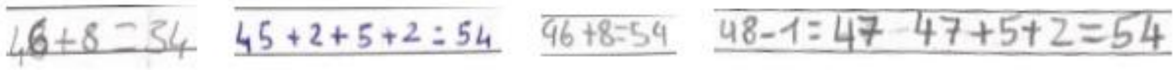
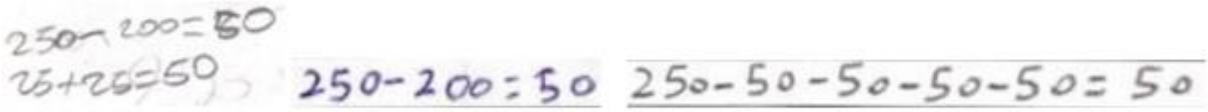


Figure 7 - Third task solutions (one group did not respond to this task)



Source: Pictures taken by the LS team

To conclude the research lesson, students were asked to answer the following question: "What did we learn today?" (request [3], as the team used this expedient to verify the goals of the lesson). The students' answers can be summarized as follows:

- "There are different ways to get a result; there is not just one solution";
- "I learned how to find different calculation strategies";
- "I learned to use the calculator better and I realized that it can be useful to me only if I know what to do";
- "I discovered that there are multiple methods for each calculation";
- "I learned how to use the calculator to do calculations without using certain buttons."

As soon as the lesson ended, the team met to discuss and analyse the research lesson. Several teaching ideas emerged from the discussion. Teachers said they would like to work on them in the future.

Firstly, the teachers reported that a group of students had difficulties with the first activity: they could not complete the second table (on the right in fig. 4). The researcher thus drew attention to the teachers' habit of proposing to the students' activities that do not involve backward processes. The researcher encouraged teachers to propose also tasks on backward reasoning. Examples are the "fill in the blanks" such as: '...+45= 78', where students cannot simply apply a known procedure to solve the task but have to reason about the missing quantity to get to 78 starting from 45.

Later, the teachers analysed the second activity and realised that in the planning stage they had not accurately defined where and how students should report the work done in groups, hindering the efficacy of class discussion (request [1]). The spokesperson of the first group wrote the group's reasoning in the centre of the whiteboard, forcing subsequent speakers to erase it in order to have space for writing their solutions. The teachers argued that, during class discussion, it would be better to divide the board *a priori* into four parts to allow each spokesperson to have their own space to write. Also, having all four solutions written on the board at the same time would have allowed the class to compare them to find similarities and differences (request [2], materials). A second issue emerged regarding the way solution strategies were shared in the class. Each group sequentially presented the reasoning carried out for the three tasks in the second activity, and then passed the turn to the next group. According to the teachers, it was complicated for the pupils to simultaneously follow the strategies applied on the three different problems. The teachers said that it would have been more productive to first reason about the strategies implemented by all four groups on the first task (47*3, use of "*" button hindered), then on the second (47+7, use of "7" button hindered) and finally on the third (250:5, use of ":" button hindered). The teachers then realized the excessive difficulty of this third task. During the research lesson, cognitive conflict emerged due to the presence of too many "5s" within this calculation: the 5 in the dividend (250), the 5 as divisor, and the 5 in the result (50). The students were confused, no longer understanding whether they were talking about the dividend, the divisor, or the result. During the discussion meeting, the teachers realised that in the planning phase they had not paid enough attention to the numbers to be included in the division; if they had chosen a division with all the digits different from each other, the children probably would not have encountered this difficulty (request [2], task). The students probably still would not have been successful in solving the task, given its high difficulty, but they would not have encountered the cognitive obstacle due to the presence of too many equal digits (request [2], task). The teachers also admitted that they had underestimated, or rather disregarded, the fact that this last task would not have allowed the children to visualise the result of the computation, albeit performed in a different manner. The meaning of the division conveyed by this task was in fact that of "by emptying" and not that of "inverse operation of multiplication", since the last interpretation would have been possible only by knowing in advance the result of the calculation.

The discussion among teachers about students' difficulties, moreover, highlighted the need to work differently on the properties of operations, which are often presented by teachers in a procedural way without a genuine reflection on their meaning. Internalising the properties of operations is also useful for speeding up mental calculation (request [4]).

Finally, among the team's reflections that emerged from observers' note and students' protocols (see figs. 5, 6 and 7, where students probably used the sign "=" with a procedural meaning) there was the need to work on the meaning of the "=" sign, even before introducing operations in class. Teachers realised that they had not invested enough in getting students to internalise its relational meaning. The discussion highlighted that in Italy, in the team's view, mathematics teaching in the early years makes students accustomed to writings like a+b=c, where the "=" sign is preceded by whatever operation and followed only by its result, and an easily engendered misconception is that it has a left-to-right reading order. The goal that has emerged from the team is to accustom students to other writings, such as c=a+b or c+d=a+b, from the early years of primary school (request [4]).

Concerning team's professional development, teachers acknowledged that LS allowed them first and foremost to develop strong reflection on their mathematical content knowledge. Teachers attested that they were not used to discuss about mathematical meanings and contents actually brought (contextualized) into the classroom (requests [1] and [4]). Secondly, in their words, LS allowed teachers to experience "a joint and co-responsible planning", where they could first-hand experience what it means to plan in detail. Indeed, the pilot teacher said: "Slowly I "got into the mood" of LS when I realized that I could not make some choices on my own. Many things I mistakenly thought I could decide on my own (student group settings, whether observers can intervene, the role of support teachers – request [5], related to teaching freedom), or improvise them (in what words to present the activities to the students), and instead we have to agree and especially justify them – i.e., declare our educational intentions – together" (request [4]). Finally, from further reflection on their experience, the participants retained extremely useful the role of observers (multiple observation – request [6]) during the research lesson. They decided to revise the way of administering the school-assessment tests by hypothesizing the possibility of the presence of observers, in order to detect not only the correctness of the results, but also the processes enacted by the students.

Third example: LS on problem decoding and solving (8th grade)

***Context*:** This LS was developed in a grade-8 class. It was composed of 17 students, 10 boys and 7 girls, five of whom certified with Mathematics Learning Disabilities (MLD).

The class in general is described by the two class teachers (two out of four in the team) as low-medium level. The teachers indicate only three students as high level in mathematics. The LS was carried out in December 2021, which, in Italy, corresponds to the end of the first half of the school year.

LS team**:** The LS team was composed by 4 teachers and 1 didactician and facilitator. All four teachers taught science and mathematics. Two of them worked in the class where the research lesson was implemented, and one of them was the pilot teacher.

A priori analysis

Aim: The aim of the research lesson – as quoted literally from Lesson Plan – was "to make you [the students] translate the text of a problem into a universal language, the language of Mathematics, that can be understood by anyone (even a foreigner), without having to go back to the original text. Try to avoid using vocabulary, prepositions, verbs, but only a mathematical language that can be understood even by those who do not speak your language" (request [2]). Therefore, the final purpose was to help students reflect on their own communication and interpretative skills, among peers, in decoding and solving mathematical problems (requests [3] and [4]).

Main idea: The research lesson was part of an introduction to literal calculations contextualised in consolidation of problem solving, text comprehension and problem decoding in Euclidean geometry and algebra (request [3]) in a mathematics laboratory setting (request [1]). The team proposed this activity to observe the students dealing with their competence in understanding, decoding and coding a text of a problem, recognising these competences as critical. Students' manipulation of (pre-)algebraic symbolism often remains anchored to algorithmic 'as-told-by-the-teacher' and meaningless resolution procedures (request [4]).

Description of the ILS

The prerequisites for the research lesson are students' knowledge of the concepts of (1) perimeter and surface area of plane figures, (2) the Pythagorean theorem and (3) basic hints of literal calculations with integer coefficients. The teachers are also aware that, when solving problems (request [1]), Italian students are accustomed to extrapolating the data and explaining the process in the "classical" schematic formulation (with the following key words, identifying the steps of the process): (identify the) *data* - (produce a) *drawing* - (identify the) *question* / *find* (the equation) *and solve* (it)/ (identify the) *unknown* followed by (report the) *solution and* (write the) *answer*.

Since the students in this class were not accustomed to working with peers, efforts were made to encourage group work and cooperative learning (requests [1] and [5]). In particular, some didactic activities were carried out prior to the research lesson: group activities on geometric problems and lessons on introducing literal calculations with integer numbers. Following the research lesson, throughout the course of the year, activities similar to the one carried out in the research lesson, with problems on solid geometry, on the circle and algebra with the literal calculations and equations, are planned in the class teaching plan (request [3]).

For the research lesson, the class was divided into groups of 3 students each. Each group is assigned one of the 7 problems prepared in advance by the team (see Table 2 – request [2], task). The problems were invented from scratch by the teachers and designed as "basic", "intermediate" and "advanced" for the different levels of the students (requests [2] and [5]). One group of students will have to work to pass on to another group all the information needed to solve the assigned problem, using only mathematical language (request [2]).

Table 2 - The 7 problems for the research lesson

Geometric content	
BASIC	Roberto and Elena compete in a race around a rectangular field measuring 40m * 30m. Starting from one corner, they have to reach the opposite corner. Roberto runs along the edge of the field, while Elena runs along the diagonal of the field. How many metres does Elena run?
INTERMEDIATE	Roberto and Elena compete in a running race around a rectangular field measuring 40m X 30m. Starting from one corner, they have to reach the opposite corner. Roberto runs along the edge of the field, while Elena runs along the diagonal of the field. How many extra metres does Roberto have to run?
ADVANCED	At the entrance to a playground, a rectangular flowerbed of dimensions 8 m and 6 m has been created. Inside the flowerbed is placed a clock inscribed in a rhombus, the latter constructed by joining the midpoints of the sides of the rectangle. How much could the diameter of the clock measure?
Algebraic content	
BASIC	In the 3B classroom there are 16 desks and 20 chairs. For an activity, 5 desks and 7 chairs are taken to the basement (where unused furnitures are). At the end of the activity, 7 desks and 4 chairs are returned to 3B. How many desks and chairs are there now in 3B?
INTERMEDIATE	In the 3B classroom there are 16 desks and 20 chairs. For an activity, 5 desks and 7 chairs are taken to the basement. At the end of the activity, 7 desks and 4 chairs are returned to 3B. How many desks and chairs are there now in 3B? How many tennis balls would be needed to silence the chairs in the classroom?
ADVANCED	In the 3B classroom there are 16 desks and 20 chairs. For an activity, 5 desks and 7 chairs are taken to the basement. At the end of the activity, 7 desks and 4 chairs are returned to 3B. How many desks and chairs are there now in 3B? How many tennis balls would be needed to silence the chairs and desks in the classroom? What needs to be done to return to the initial situation?
Extra problem for groups finishing before the set time	

INTERMEDIATE	Marco wants to prepare a chocolate cake for his birthday. The recipe says that 600 g of chocolate is needed. In the supermarket they sell chocolate bars of 250 g each. If each bar consists of 10 squares, how many squares of chocolate does Marco need to make the cake?

Source: Elaboration by the LS team; Translation by the authors

In a second phase of the research lesson, the receiving group will have to solve the problem using only the information provided (i.e., they will not be able to read the original text of the problem – request [2]). After solving the problem, each group returns the solved problem to the sender group.

In the third phase, the two groups come together to discuss the strategies used to translate the problem and understand the data (request [2]). Between phases 1 and 2, the decoded problem can be returned to the sender if it is not understandable and solvable; the first group is not necessarily required to solve the problem, but only to encode the text (request [2]).

For the concluding phase, a large-group discussion on the activity was planned (request [1]) in order to make explicit the teachers' aim for the activity: i.e., to focus on the decoding and formalisation part, rather than on the problem-solving part (request [3]). However, this phase has never been carried out, due to lack of time.

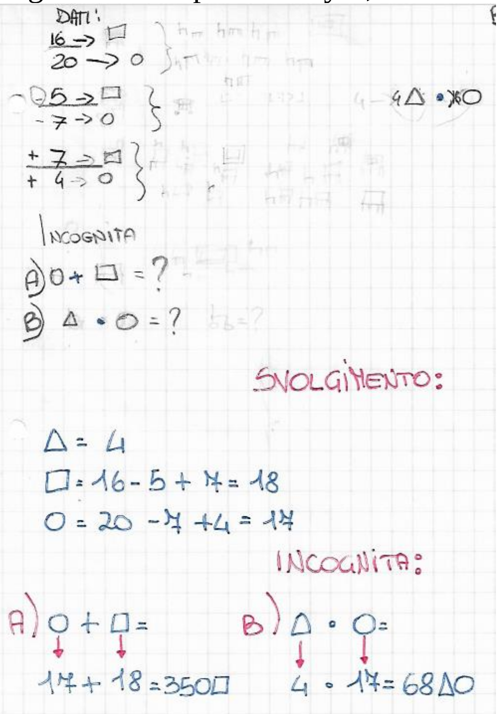
In order to collect useful data to analyse and discuss the development of the research lesson, the team decided to film the lesson through a tablet camera hold by the didactician, and to collect the notes taken on the spot by the 3 observer teachers present in the classroom (request [6]).

The discussion meeting was held the week following the research lesson. The team met again after watching the video recording. After the pilot teacher's account of first impressions, it is from the videos that the discussion arose.

For reasons of space, we report here only one example of a reflection made from a video excerpt. It was chosen because it is particularly significant to the aim of the research lesson: it concerns students' pre-algebraic competence and their spontaneous meta-reflective ability.

M.:So we know that the triangle “times” the circle...
B.: equals...
M.:...it would have been 4 times 17, right?
M: Ok.
S: So let’s try to do that.
M.:Ah, I used your [calculator], sorry. 4 times 17.
S.: and M.: 68.
S.: But what is 68? [pause] Squares... well, triangle plus circle.
B.: Triangle and circle.
M.:Eh, we have to check...
M.:It will be... it’s possible that it’s like a, I don’t know, 2*ab* squared...
S.: Ah.
M.:either a calculation or a literal thing.
S.: In my opinion, yes.
M.: 68.... So let’s put...
S.: ...triangle and circle.
M.:triangle-circle.
M.:Then we have... circle plus square. Well, we haven’t made the squares yet.
M.:A square must be equal to... 16
S.: ok, um... minus 5... [M repeats and types]
S.: plus 7. [M repeats and types]
M.:And comes 18.
S.: 18 plus 17.
M.:In my opinion I wrote it wrong... Or maybe I made a mistake: you told me to do “less” and I put “plus”.
B.: No, 18 is right. 18 is right!
M.: Is 18 right? Okay! So I wrote right.
S.: So the square...
M.:18... therefore we have the circle... So, it will then be... circle plus square...
M.:which is equal to... The square did we say 18... and 17?
S.: yyyesss...
M.:So it’s 35 circle-squares...

Figure 8 - The protocol by S, M and B.



Source: Picture taken by the authors

The previous is an excerpt from one of the student groups' discussions, while reflecting on the coded text of the intermediate level algebraic content problem. The group consists of two students considered low level (S and M; M also has MLD-certification) and one middle level (B) (request [5]). Fig. 8 shows the protocol of this group: the first part (in black) shows the coded text sent by the counterpart sending-group, and the second part (in blue and red) shows the solution proposed by the observed group (receiving-group), according to their interpretation of the problem.

This excerpt was particularly interesting because we can see the students group wondering how to determine the result of their "triangles times circles" calculation. They knew that they had to determine 68 with a label, and that this had to have something to do with circles and triangles. It occurred to M that it could be like 2*ab* squared, as in the computation of the double product of the "binomial square". She meant that, as in the literal calculation in the case of a product (2 "times" *a* "times" *b* = 2*ab*), here too the literal part of 68 must be added and, in this case, it is "4 triangles times 17 circles = 68 triangles-circles". However, the same process was repeated by the students in the case of the sum of circles and squares, heedless of the difference in literal calculation between sum and multiplication. Moreover, this determination would be nonsense if contextualised in the problem-situation of chairs to be soundproofed with tennis balls. For the LS team, having been unable to realise the mathematical discussion, the challenge of using this students' 'misconception' to address the meaning of the literal calculus persisted (request [4]). What do you obtain by multiplying 4 tennis balls (triangles), i.e. one for each chair leg, by the number of chairs (circles); and then what are those 68 objects you get and how are they determined. Or what it means to add desks and chairs together, as well as what it means to add 17*a* and 18*b*.

The teachers realised that, without LS, i.e. without this detailed observation of the videos (request [6]) in which the students discuss, this opportunity would have been lost. Literal calculation would have been taught as a set of procedures, unrelated to the students' experience (requests [1] and [2]). After this analysis, the pilot teacher instead said that she would start from this group misconception to address the issue within the class (request [4]).

Fourth example: LS on origami geometry (9th grade)

The results of the surveys mentioned in the Introduction were particularly taken into consideration when preparing a professional development meeting with the teachers participating in the professional development course there referred to. As didacticians, we explicated the meaning of the term “lesson” within LS and ILS (in the researchers’ notes to the presentation we read “LESSON as something that takes place in a specific place and time frame”), we recalled the ministerial meaning given to the term “lesson” (as a 1-hour segment), specified at several points that the research lesson would take one hour, and that teachers would have to design a one-hour lesson.

Context: This LS was developed in a grade-9 class, composed of 28 students, two of them with MLD. Only few pupils are considered “high achievers”. The class is used to work in a mathematics laboratory setting.

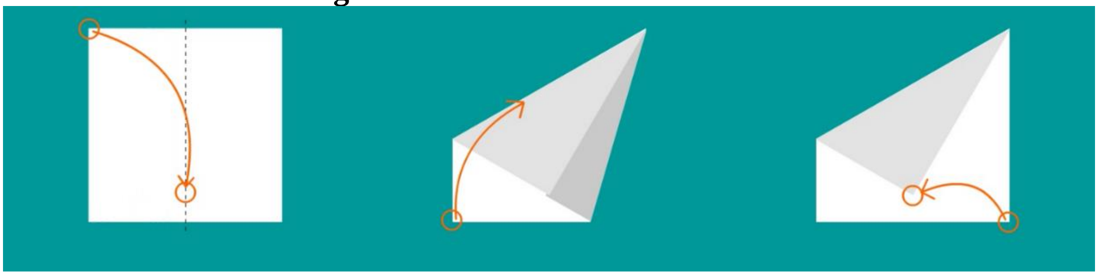
LS team: The team consisted of 4 teachers of a scientific-oriented high school with 18 to 35 years of teaching experience. None had experience as researcher or teacher educator. The other team member was a PhD Student, the didactician. The teachers declared that this was their first time collaborating with others to plan something other than the curriculum for the year, which is decided during a meeting of all mathematics teachers of the school at the beginning of the school year. One of them said, “I feel positive about knowing others’ experiences, but I also feel some apprehension when they share methodologies without a tradition”.

A priori analysis

Aim: The teachers planned an activity for 9th grade on synthetic Geometry, with triangles as main topic. Their main aim was to develop students’ abilities around discussion, conjecture, argumentation and proof (request [4]).

Main idea: The main point of the activity is to have the students fold a squared piece of paper following a set of instructions to obtain a triangle (Figure 9), and use geometry rules to classify it according to the angles or sides (request [2], task). Citing the Lesson Plan: “folding paper to create figures is a laboratory of intuitive geometry; the students are active on different levels, in a stimulating context to experience and discover mathematical objects. Moreover, paper folding has high learning potential, as it involves visual, manual and thought capabilities; finally, it is accessible and allows one to focus on concepts” (request [2], materials).

Figure 9 - Instructions from the video



Source: Snapshots from the video

The activity is divided into three 1-hour lessons (request [3]): in the first one, the "axioms of origami geometry" are introduced, simple loci are constructed (e.g., axis, bisector); in the second one, the students work in small groups to fold the aforementioned squared piece of paper, classify the obtained triangle and discuss their results with the whole class (request [2], task); in the third lesson, students use GeoGebra to reproduce the folding. The second lesson (i.e., the research lesson) was to "have the students come up with conjectures (whether correct or incorrect) and propose arguments to validate them" (request [3]); the teachers decided to observe "the dynamics of homogenous groups, which for us is an unusual setting" and "the ability of selected students to expose and discuss their hypotheses" (request [6]). In the first 10 minutes, the teacher provided the students with the piece of paper, and instructed the students on the folding. To facilitate the students, a video – showing the folding process (Figure 9) – was shown on the multimedia whiteboard of the classroom for the first 30 minutes of the lesson (request [2], materials). The students had 20 minutes to work in groups, solve the task and justify their solution. Then, each group was supposed to briefly share and justify their results (10 minutes overall), followed by 10 minutes of whole-classroom discussion of the results (request [1]). As the sharing took more than 10 minutes, the whole-classroom discussion did not take place.

Description of the ILS

The LS cycle took place over four meetings (at least 2 hours each) of the whole team (audio-recorded), plus the research lesson. The first meeting was to share more insights on LS; the second to decide the long-term goals of the LS and define the teaching activity; the third to plan the research lesson; the fourth to discuss the research lesson. Between the second and third meeting, and the third meeting and the research lesson, two undocumented meetings happened between teachers only, in which they defined details of the teaching activity.

Before the first meeting, the teachers emailed to the didactician a document with questions about LS, discussed during the first meeting. The main theme was "time", in particular the time dedicated to the research lesson: "[we] are used to planning lessons of four or five hours. In one hour, it is not possible to get interesting results with the students (request [3]). What if this hour does not go as planned?". During the meeting, a teacher commented: "I know that my lesson will take six hours, and I know the goal of those six hours, but I do not know – explicitly – what is the aim of each of those six hours" (requests [3] and [4]). The didactician noticed that the term "lesson" still had different meaning for him and the teachers. So he provided two reasons to justify the choice of the "1-hour research lesson": by stressing the distinction between "*activity planning* […] something teachers are used to" and "*lesson planning* […] something new which could shed light on some aspect of their work that they did not reflect about"; and by practical reasons (e.g.,: "a 1-hour lesson is easier to prepare […] it is easier to keep the focus on our goals"; or "we want to share our work with other teachers, it is good to also have a standard format").

We can notice that the teachers adopted the shared language. In the second meeting, they noticed that "achieving our goal in one lesson, even in one activity, is impossible. For now, we should assess their argumentation abilities" (request [3]). To this goal, the didactician suggested three different tasks: folding a triangle and categorising it; understanding if cutting a straw in three pieces always gives a triangle; folding a parabola to understand its properties. The teachers mainly discussed the first and third tasks. Finally, they chose the first one to hold the research lesson with grade-9 pupils. The pilot teacher was also chosen. With no further input from the didactician, they structured the activity over three lessons, and the pilot teacher had a focal role in choosing this structure. The goal of the first lesson was "to prepare the setting in which they will work and get them used to manipulating the paper", while the goal of the third lesson was "to institutionalize the discussions and knowledge from the second lesson" (request [3]). Doing so, they showed that the difference between "activity" and "lesson" was clearer than before.

During the third meeting, the teachers' worries about the time available for the research lesson emerged again. The pilot teacher seemed the most worried: "one hour is not enough […] we are not ready […] we should move the date further". This brought some tension in the team, as the date for the research lesson had been decided in the first meeting, and the other members had accommodated their schedule accordingly. Even so, the planning went smoothly: the teachers focused on planning the lesson and the task according to the *Indicazioni Nazionali* (only slightly referenced in the previous meetings), reading them several times during the planning. In particular, the *Indicazioni Nazionali* recommends that the students: "know how to support their argument and can listen to and critically evaluate the others' arguments; acquire logical rigour when reasoning, identifying problems and finding possible solutions". Therefore, the goal of the lesson was established as "to have the students come up with conjectures (whether correct or incorrect) and propose arguments to validate them" (request [3]). The teachers noticed that the activity fit well with another recommendation in the *Indicazioni Nazionali*: "the Euclidean approach will not be treated as merely axiomatic". It was also decided, following a suggestion from the didactician, to organize the pupils in homogenous-by-level groups. The teachers were not used to this way of grouping, as they thought that the low-level ones would have felt uncomfortable due to their inability to contribute to the knowledge. After some discussion, they decided to test the setting (request [4]). The pilot teacher was especially worried about the two students with MLD, and proposed to focus on their performance during the lesson (request [5]). The goal of the observation was twofold: "the dynamics of homogenous groups" and "the ability of selected students to expose and discuss their hypotheses" (request [5] and [6]).

The fourth and final meeting was the same day of the research lesson. It lasted two hours and thirty minutes: two hours to discuss the lesson and thirty minutes to share impressions on LS. The meeting began with an exposition from the pilot teacher, then the team discussed in no particular order. Concerning the goals of the lesson, all teachers considered them only partially achieved: all the students were, in fact, able to produce some level of argumentation and expose to the classroom, but they were not able to listen to others' arguments. The pilot teacher was surprised by the attitude of the students when presenting their argumentation: they were speaking to the teacher, not to their peers. They concluded that these students probably believed that every exposition to the classroom was, in fact, an oral test from the teacher (request [4] and [5]). The pilot teacher noted that he usually collects ideas from the pupils without focusing on "who contributes", so the pupils are also not used to being all in charge of contributing (request [4]). Concerning the goals of the observation, the teachers were surprised by the performance of the homogenous groups: the pilot teacher noted that "the low-level students could not contribute much when presenting to the whole class, but they were all very focused inside the group while, usually, they lose focus after few minutes"; he also noted that "she [a girl who had problems integrating into the class] was able to take the lead of her [high-level] group, I think that she felt proud of being part of that group"; finally, he was surprised that the two middle-level groups were able to provide the more complete answers to the task, whether the high-level group could not. He said that this kind of grouping allowed “some usually-hidden dynamics to emerge” and that “I found out some possibilities for working in the classroom that I had forgotten about, or maybe failed me in the past and I decided that they would never work” (request [4] and [5]). The team agreed with him, albeit one teacher said: “Homogenous groups probably would not work with my pupils” (request [5]).

Finally, when discussing LS, all the teachers were impressed by the results of the collaboration: “I loved these moments where we meet and share different experiences”, “it is more productive”, “we do not usually think about such short-term goals, but they are also very important”. On this point, the teachers discussed that “we unconsciously know which are the goals of the lesson, but if we make them explicit they also feel more real, so we think about them more carefully” (request [3]). They also discussed the difficulties that they had in predicting students’ reactions: “I think every teacher believes that they will be able to answer whatever question comes from the students, while they are not”. Another teacher commented, “I particularly fail to predict the ‘silly’ questions, so I do not know how to react”. The main issue, however, was time management, on different levels: the teachers noted that LS cycles should be planned at the beginning of the school year so that it can be better implemented (e.g., more time to articulate the topic in more detail, over the course of more lessons); they also thought that one hour for the research lesson is not enough, using the word “cage” to refer to how they felt in this regard.

Conclusions

In this paper we have given four examples from all the different school levels (grades 1 and 5 for primary school, grade 8 for lower secondary school and grade 9 for upper secondary school) that we think emblematic of what happens when the Italian teachers are introduced to LS as a tool for teacher education. They concretely illustrate some typical issues that teachers could face in such a teacher education process and that are at the basis of our (Italian) team theoretical elaboration of LS[5]. Our activities with teachers brought us, as didacticians, to work with the notion of Cultural Transposition (CT) in order to elaborate what we called ILS (Italian Lesson Study). As pointed out in the Introduction, CT allows to use foreign teaching practices into a different cultural school context, using them consistently but also dialectically retaining some of the related conflictual aspects. In this process, the didacticians are responsible for facilitating the teachers’ work on the specific

[5] As highlighted above, apart some rare singular activities, e.g., that of R. Capone (Capone et al., in print), who however work with us on LS, our research team is the only one working systematically in Italy on LS.

identified conflictual aspects, but the validity of such a teacher education process consists in the general chosen design of our experiments: it is a feasible scientific and ethical approach that allows to develop an Italian effective way to LS. In fact, it avoids improvised or not shared changes and it can represent a real improvement of teachers' professional development, with stable effects on their didactical practices and beliefs. In this final section we sketch a compact picture of some of the results got through our 4 examples using the analysis tools we introduced above, namely the list of the 8 *cultural implicit factors* in the transposition of the Lesson Study, and the *Requests* used to analyse the design choices and the related activities.

First, we reconsider the conflictual elements pointed out in the *cultural implicit factors*' list. We have verified that these elements are active in the CT process, and we now distinguish two levels[6] among them: (i) a *general background* level, that is a context that influence general didactical attitudes and beliefs of teachers with respect to their mathematical teaching activities when working on LS (factors 1, 3, 4 and 5); (ii) a *specific foreground* level, consequence of the previous one, which determines the effective teachers' practices in their activities during the ILS cycles (factors 2, 6, 7 and 8). It is within this second level that we have observed the main changes in teachers' practical and theoretical approach to their mathematics teaching and professionalism, when in interaction with the LS. On the other hand, with regard to the background level, it is where teachers gain awareness with respect to their practices. For example, the *Indicazioni Nazionali* (factor 1) do not prescribe that teachers develop detailed didactical plans, and the constitutionalized freedom of teaching (factor 3) make it so that the Ministry does not provides a pre-organised succession of contents to be taught nor indications of educational choices; adding to this, the habit in the school is that mathematics teachers of a school meet a couple of times during the year to elaborate the general content of their courses, listing only the main mathematical topics to teach without entering into many didactical details (factors 2, 6 and 7). This causes teachers to experience loneliness in planning their teaching and to perceive lax design as an expression of their professionalism, a proficiency in accommodating to situations. As said above, the combination of these factors resulted in the *requests* we identified as necessary to construct LS coherently with the Italian didactic culture: indeed, in this context, requests 2,

[6] This framework could be properly described within Chevallard's Anthropological Theory of Didactics (Bosch & Gascón, 2006; Arzarello et al., 2014), distinguishing between the logos and the praxis in teachers' meta-didactical praxeologies, articulated at different levels of didactical co-determination (Chevallard, 2002). It is beyond the descriptive aim of this paper to elaborate this theoretical analysis.

3 and 4 become matter of elaboration of new meanings (which we have tried to describe in detail in each example, both in terms of mathematical and pedagogical knowledge). Despite the tensions that they bring in the teachers' minds, they positively induce teachers to discover 'unthought' parts of their work. What at the beginning appears as an initial 'misunderstanding', with time is elaborated in a positive mood, possibly because of the discussion in the group, or with a proper approach from the didactician and/or the facilitator.

Before sketching some more specific comment on the four groups' examples, it is now important to add some observations on the 6 *requests* (Table 1). Some very simple statistical elaborations allow a finer within- and between-comparative analysis of the *requests* (r[n]) with respect to the school grade (labelled here as GR, and numbered according to school grade).

Table 3 - percentages of *requests* (r[n]), normalised to the grade **(a)** or to the request **(b)**

	r[1]	r[2]	r[3]	r[4]	r[5]	r[6]	
GR1	16	28	20	24	8	4	100
GR5	12	31	19	19	12	8	100
GR8	20	28	16	16	12	8	100
GR9	4	15	31	23	19	8	100
	13	25	22	21	13	7	

	r[1]	r[2]	r[3]	r[4]	r[5]	r[6]	
GR1	36	32	26	32	17	17	25
GR5	27	36	26	26	25	33	25
GR8	27	14	5	11	17	17	25
GR9	9	18	42	32	42	33	25
	10	10	10	10	10	10	
	0	0	0	0	0	0	

Source: Elaboration by the authors

Table 4 - Pearson's correlation coefficient between requests

	r[1]	r[2]	r[3]	r[4]	r[5]	r[6]
r[1]	1,00	0,73	-0,96	-0,66	-0,81	-0,29
r[2]		1,00	-0,89	-0,50	-0,84	-0,19
r[3]			1,00	0,70	0,84	0,19
r[4]				1,00	0,21	-0,52
r[5]					1,00	0,66
r[6]						1,00

Source: Elaboration by the authors

Table 3a indicates the percentages of Requests used in each analysis, (horizontally) normalised with respect to each grade, with the percentage of incidence of each request in all grades out of the total number of requests (last row in blue). Table 3b, instead, contains the same data, but (vertically) normalised with respect to each Request: they show (last column in blue – percentage of incidence) that the four descriptions share a common analysis modality with respect to the different Requests used in it. Table 4 contains the correlations between the Requests presence with respect to the different groups (i.e., derived from the columns of Table 3a). A positive correlation (r1#r2,r3#r4,r3#r5) indicates some probable complementary affinity between them: for example, the attention to hands-on activities (request [1]) can imply a corresponding attention to the use of materials (request [2]); as well, it is easy that requests concerning a lesson short term objectives (request [3]) are related with teacher's reflection on educational intentionality (request [4]) and the analysis of the classroom context (request [5]). On the contrary, negative values (r1#r3, r1#r4, r1#r5, r2#r3, r1#r4, r2#r5, r4#r6) show that these requests are relatively independent from each other: e.g., the consideration of hands-on activities (request [1]) may pose different problems, i.e. concerning content features, from those put forward by requests concerning the classroom context (request [5]), for example from the point of view of inclusiveness.

After this cross-comparative analysis between the factors and the requests used in our analysis, which sketches the validity and limits of our lens, let us now come to some of the main specific ILS features, which this lens has allowed us to focus in the four examples of the study:

- In GR1, perceived differences with respect to the Eastern LS experiences produce a constructive CT. Teachers' reflection on Chinese children behaviour generates a specific attention on the sequence of tasks in an inclusive perspective: the difficulties of managing the discussion bring to double possibilities of designing a research lesson;
- In GR5, the stimulus to a bigger co-responsibility in planning and an intense attention to cooperative interactions produces a new sensitivity to the importance of small details in the design and explication of educational intentionalities, which was missing before, thus generating also a change in teachers' mathematical content knowledge.
- In GR8, a fresh attention to the usefulness of observation modalities in the classroom, in particular of students' group work, can be observed: the focus on mathematical content explodes in the fine observation of a few seconds of dialogue between students and their textual signs, this suggests the need for a new observational posture to teachers.

- In GR9 the decision to work with ‘groups of levels’ generated a higher attention to an unusual activation of observation practices in the class and to a similarly unusual collaborative spirit of co-working among teachers.
- The issue of inclusivity was an unexpected element particularly in GR9, where teaching design is not often linked to the classroom context. The GR9 LS team found ‘unthought’ results from the group-levels: contrary to their expectations, low achievers showed quite good productions, while the best results were got by middle level achievers, since the high-level ones did not perform as teachers expected. All these aspects contributed to the ‘unthought’ phenomenon hinted above: it appeared in different groups and generated processes of change in teachers’ beliefs and practices.
- A main issue of perplexity in many teams concerned the time, which deeply contrasted with the beliefs and practices of most groups. A typical comment, depicting the initial scepticism shared by all groups on the actual feasibility of LS, is the quoted comments of GR9 teachers: “[we] are used to planning lessons of four or five hours. In one hour, it is not possible to get interesting results with the students”. In the final comments they wrote that the issue of time in LS represented a ‘cage’ for them. In this sense, it remains for them one of the most delicate issues not completely solved in the CT. This creates in particular a remarkable difference between secondary school, especially its higher years, and primary school, where this issue triggered productive reflections about the issue of time in teaching (Arzarello et al., 2022). In any case, it remains an open question for researchers. From current investigations it seems that, in higher secondary school, starting the LS design from a main focus on the specific mathematics topics can help in developing an approach to the ‘time issue’ that seems an effective CT of the way ILS can structure the design of tasks according to a shorter time structured model.

Other aspects we observed were induced by a common mood of perceiving the experience with LS as something of unknown and possibly even mysterious. For these reasons all the groups strongly concentrated on the mathematical content to choose for the LS design: all of them considered it something very important, on whose didactical approach they thought to have a good knowledge and strong beliefs and possibly could produce innovative aspects (in particular with respect to requests [1] and [2]) without entering too much into the ‘unknown’ territory of the LS, but elaborating the same a coherent task design.

Because of that, the groups generally decided to design tasks, where a possible innovative mathematical content was embedded in new practices. In a sense, the confidence with the mathematical content and the hands-on designed activities allowed the teachers to consider with an open and curious mind the unusual aspects of their didactical activities. These first produced (for them) surprising positive experiences, then a reconsideration of the unusual teaching experiences, and in the end, sometimes, changes in the teaching practices with the consequent elaboration of a fresh terminology ('lesson', 'observation', 'co-design').

Concluding this summary of the ILS with lights and shadows we point out that to deepen the analysis of the CT processes in ILS we think it will be productive to enlarge the analysis developed in Arzarello et al. (2022) through the hybridization construct, also elaborating an investigation focusing on the dynamics of background and foreground cultural implicit factors through the Chevallard's lens of the levels of co-determination (Chevallard, 2002). As pointed out by Winsløw (2011), they can be useful for considering the institutional levels beyond the classroom: from the curricular materials and regulations, to school pedagogy and policies, as well as wider cultural conditions that surround the school and its students. This is the natural landscape, where ILS can be better understood and properly defined.

References

Arzarello et al. (Eds.). (2012). **m@t.abel: Mathematics for students on the threshold of the third millennium**. INDIRE/ANSAS.

Arzarello, F., Funghi, S., Manolino, C., Ramploud, A., & Bartolini Bussi, M. G. (2022). Networking Hybridizations within the Semiosphere: a research trajectory for the Cultural Transposition of the Chinese Lesson Study within a Western context. **International Journal for Lesson & Learning Studies, 11**(4), 331–343.

Arzarello, F., Robutti, O., Sabena, C., Cusi, A., Garuti, R., Malara, N., & Martignone, F. (2014). Meta-Didactical Transposition: A Theoretical Model for Teacher Education Programmes. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), **The Mathematics Teacher in the Digital Era** (Vol. 2, pp. 347–372). Springer Netherlands. https://doi.org/10.1007/978-94-007-4638-1_15

Bartolini Bussi, M. G. (1996). Mathematical discussion and perspective drawing in primary school: To Giovanni Prodi on occasion of his 70th birthday. **Educational Studies in Mathematics, 31**(1–2), 11–41.

Bartolini Bussi, M. G., & Martignone, F. (2013). Cultural issues in the communication of research on mathematics education. **For the Learning of Mathematics, 33**(1), 2–8.

Bartolini Bussi, M. G., Sun, X., & Ramploud, A. (2014). A dialogue between cultures about task design for primary school. In C. Margolinas (Ed.), **Proceedings of the ICMI Study 22** (pp. 549–558). Oxford.

Bartolini Bussi, M. G., Bertolini, C., Ramploud, A., & Sun, X. (2017). Cultural transposition of Chinese lesson study to Italy: An exploratory study on fractions in a fourth-grade classroom. **International Journal for Lesson and Learning Studies, 6**(4), 380–395.

Bosch, M., & Gascón, J. (2006). Twenty-five years of the didactic transposition. **ICMI Bulletin, 58**, 51–65.

Braga, P., & Tosi, P. (1998). L'osservazione. In S. Mantovani (Ed.), **La ricerca sul campo in educazione. I metodi qualitativi**. Milano: Bruno Mondadori.

Capone, R., Adesso, M. G., Manolino, C., Minisola, R., Robutti, O. (in print). Culturally Crafted Lesson Study to Improve Teachers' Professional Development in Mathematics: A Case Study in Italian Secondary School. **Journal of Mathematics Teacher Education.**

Cardarello, R. (2016). L'osservazione in classe. In Ufficio scolastico regionale (Ed.), **Essere docenti in Emilia Romagna 2015-16** (pp. 51–55). Napoli: Tecnodid.

Castelnuovo, E. (2008). **L'officina matematica**. Edizione La Meridiana, Molfetta.

Chevallard, Y. (2002). Organiser l'étude. 3. Écologie & regulation. In J. L. Dorier (Ed.), **Act es de la 11e École d'Été de Didactique des Mathématiques** (pp. 41–56). La Pensée Sauvage.

Jaworski, B., & Potari, D. (2021). Implementation of a developmental model of teachers' and didacticians' learning through inquiry: Design, operationalisation and outcomes. **ZDM, 53**(5), 1073–1084. https://doi.org/10.1007/s11858-021-01290-x

Jullien, F. (2006). **Si parler va sans dire. Du logos et d'autres ressources.** Paris: Edition du Seuil.

Manolino, C. (2021). The Semiosphere lens to look at Lesson Study practices in their cultural context: A case study. In M. Inprasitha, N. Changsri, & N. Boonsena (Eds.), **Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education** (pp. 263–272).

Mellone, M., & Ramploud, A. (2015). Additive structure: An educational experience of cultural transposition. In Sun X., Kaur B. and Novotná N. (Eds.), **Proceedings of the ICMI Study 23** (pp. 567– 574). China, Macau: University of Macau.

Mellone, M., Ramploud, A., Di Paola, B., & Martignone, F. (2019). Cultural transposition: Italian didactic experiences inspired by Chinese and Russian perspectives on whole number arithmetic. **ZDM, 51**, 199–212.

Mellone, M., Ramploud, A., & Carotenuto, G. (2021). An experience of cultural transposition of the El'konin-Davydov curriculum. **Educational Studies in Mathematics, 106**, 379–396.

Minisola, R. (2016). **Insegnanti di Matematica che lavorano in collaborazione: panoramica internazionale e contesto italiano** [Master's dissertation]. University of Turin.

Minisola, R., & Manolino, C. (2022). Teachers' professional development: a cultural matter. How to describe cultural contexts? In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), **Proceedings of the Twelfth Congress of European Research in Mathematics Education** (CERME12). (pp. 3650–3657). ERME / Free University of Bozen-Bolzano.

MIUR (2012). http://www.comune.torino.it/centromultimediale/01c_I_documenti_pedagogici/documenti _Nazionali_pdf/2012_Indicazioni_Curricolo_Sc_Inf.pdf

OECD (2020), **TALIS 2018 Results (Volume II): Teachers and School Leaders as Valued Professionals**, TALIS. OECD Publishing. https://doi.org/10.1787/19cf08df-en.

Ramploud, A., Funghi, S., & Bartolini Bussi, M. G. (2022). Chinese lesson study: critical aspects of transfer from China to Italy. **International Journal for Lesson & Learning Studies, 11**(2), 147–160. ISSN: 2046-8253, https://doi.org/10.1108/IJLLS-04-2021-0031

Schiedi, A. (2021). The Montessori theory in the "No Schoolbag" model. Formativity of materials and of the educational environment. **Ricerche Di Pedagogia e Didattica. Journal of Theories and Research in Education**, 93-104 Pages. https://doi.org/10.6092/ISSN.1970-2221/12199

Venkat, H., Beckmann, S., Larsson, K., Xin, Y. P., Ramploud, A., & Chen, L. (2018). Connecting Whole Number Arithmetic Foundations to Other Parts of Mathematics: Structure and Structuring Activity. In M. G. Bartolini Bussi & X. H. Sun (Eds.), **Building the Foundation: Whole Numbers in the Primary Grades** (pp. 299–324). Springer International Publishing. https://doi.org/10.1007/978-3-319-63555-2_13

Winsløw, C. (2010). Anthropological theory of didactic phenomena: Some examples and principles of its use in the study of mathematics education. In M. Bosch, J. Gascón, A. R. Olarría, M. Artaud, A. Bronner, Y. Chevallard, G. Cirade, C. Ladage, & M. Larguier (Eds.), **Un panorama de la TAD** (pp. 117–140). Centre de Recerca Matemàtica.

Autores

Ferdinando Arzarello

Laurea in Matematica at Turin University

Specialisation in Logic at Amsterdam University

Emeritus professor at Department of Mathematics, Turin University

Mathematics Education Research Group in Turin University

ferdinando.arzarello@unito.it

https://orcid.org/0000-0002-0913-7500

Maria Giuseppina Bartolini Bussi

Master Diploma (Laurea) in Mathematics at the University of Milan

(Retired professor at) Department of Education and Human Science, Modena-Reggio Emilia University

Mathematics Education Research Group in Modena-Reggio Emilia University

Lesson Study; teacher education

mariagiuseppina.bartolini@unimore.it

https://orcid.org/0000-0002-3257-1348

Silvia Funghi
Bachelor in Mathematics, University of Pisa
Master in Mathematics, University of Pisa
Ph.D. in Humanities - Didactics of Mathematics, University of Modena and Reggio Emilia
Work institution: Department of Mathematics, University of Turin
"S. Polin" Laboratory for research on assistive technologies for teaching-learning of STEM disciplines; affective and cultural aspects in mathematics education
silvia.funghi@unito.it
https://orcid.org/0000-0001-8549-8474

Carola Manolino
Bachelor in Mathematics, University of Turin
Master in Mathematics, University of Turin
PhD in Pure and Applied Mathematics, University of Turin
Work institution: University of Turin
"S. Polin" Laboratory for research on assistive technologies for teaching-learning of STEM disciplines; Mathematics Education Research Group in Turin University
Mathematics and Visually Impairment; Lesson Study; semiotics; teacher education
c.manolino@univda.it
https://orcid.org/0000-0002-8711-4638

Riccardo Minisola
Bachelor in Mathematics, University of Cagliari
Master in Mathematics, University of Turin
PhD Student in Pure and Applied Mathematics, University of Turin
Work institution: University of Turin
Mathematics Education Research Group in Turin University; Lesson Study; teacher education
riccardo.minisola@unito.it
https://orcid.org/0000-0002-1105-4843

Alessandro Ramploud
Master in Philosophy, University of Bologna
PhD in Humanities - Didactics of Mathematics, University of Modena and Reggio Emilia
Work institution: Department of Mathematics, University of Pisa
Center of Advanced Research on Mathematics Education CARME; Cultural Transposition
alessandro.ramploud@dm.unipi.it
https://orcid.org/0000-0002-9161-4213

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