

“When you find the right task, you'll know”: design and validation of a geometric task—a case study

Juan Pablo Vargas Herrera¹  Luca Lamanna² 

Abstract

The design of a task entails a process in which several elements must be considered that play an active role in the choices that are made and address, in particular, the issue of the suitability of the task that is created. The aim of this article is to present an autoethnographic case study focused on the process followed by two teacher-researchers when designing a task for the assessment of learning related to a geometric object. In the analysis, tools of the Ontosemiotic Approach are used to identify and describe the knowledge produced in the processes of formulation, resolution and validation of the task. Adopting an ethnographic perspective, this paper presents the path followed from the genesis of an idea for a task to its validation, through a reflection on the choices made and focusing on the experience of the eureka moments identified as key in the whole process. As a result, elements of the eureka moments experienced and the trajectory followed by the teachers are described, since these elements are considered fundamental in the processes of problem solving and task design. Finally, as part of the discussion, the authors reflect on the importance of the choice or design of tasks and activities and their relationship with the affective dimension since the achievement of these types of moments creates positive beliefs and attitudes about mathematics, as well as about the students' abilities to do mathematics.

Keywords: 3D Geometry, Task-design, Teacher Competence, Ontosemiotic Approach.

“Quando você encontrar a tarefa certa, você saberá ”: design e validação de uma tarefa geométrica – um estudo de caso

Resumo

O planejamento de uma tarefa envolve um processo no qual devem ser considerados vários elementos que desempenham um papel ativo nas escolhas que são feitas e abordam, em particular, a questão da adequação da tarefa que é criada. O objetivo deste artigo é apresentar um estudo de caso autoetnográfico focado no processo seguido por dois professores-pesquisadores ao desenhar uma tarefa para a avaliação da aprendizagem relacionada a um objeto geométrico. Na análise, são utilizadas ferramentas da Abordagem Ontossemiótica para identificar e descrever o conhecimento produzido nos processos de formulação, resolução e validação da tarefa. Adotando uma perspectiva etnográfica, este artigo apresenta o caminho percorrido desde a gênese de uma ideia para uma tarefa até sua validação, por meio de uma reflexão sobre as escolhas feitas e com foco na experiência dos momentos eureka identificados como chave em todo o processo. Como resultados, são descritos elementos do momento eureka vivenciados e a trajetória seguida pelos professores, uma vez que esses elementos são considerados fundamentais nos processos de resolução de problemas e desenho de tarefas. Por fim, como parte da discussão, os autores refletem sobre a importância da escolha ou do desenho de tarefas e atividades e sua relação com a dimensão afetiva, uma vez que a obtenção desses tipos de momentos cria crenças e atitudes positivas sobre a matemática, bem como sobre as habilidades dos alunos para fazer matemática.

Palavras-chave: Geometria 3D, Design de Tarefas, Competências do Professor, Abordagem Ontossemiótica.

¹ Universidad de las Américas (UDLA), Santiago de Chile, Chile. E-mail: juvargas@udla.cl

² University of Milan (Statale), Milan, Italia. E-mail: luca.lamanna@unimi.it

"Cuando encuentres la tarea adecuada, lo sabrás ": diseño y validación de una tarea geométrica—un estudio de caso

Resumen

El diseño de una tarea implica un proceso que requiere la consideración de diversos elementos que influyen en las decisiones tomadas y que abordan, en particular, la idoneidad de la tarea creada. El objetivo de este artículo es presentar un estudio de caso autoetnográfico centrado en el proceso seguido por dos docentes-investigadores al diseñar una tarea para la evaluación del aprendizaje relacionado con un objeto geométrico. En el análisis, se utilizan herramientas del Enfoque Ontosemiótico para identificar y describir el conocimiento producido en los procesos de formulación, resolución y validación de la tarea. Desde una perspectiva etnográfica, este artículo presenta el recorrido desde la génesis de una idea para una tarea hasta su validación, a través de una reflexión sobre las decisiones tomadas y centrándose en la experiencia de los momentos eureka identificados como claves en todo el proceso. Como resultados, se describen los elementos del momento eureka vivido y la trayectoria seguida por los docentes, ya que estos elementos se consideran fundamentales en los procesos de resolución de problemas y diseño de tareas. Finalmente, como parte de la discusión, los autores reflexionan sobre la importancia de la elección o diseño de tareas y actividades y su relación con la dimensión afectiva ya que el logro de este tipo de momentos crea creencias y actitudes positivas sobre las matemáticas, así como sobre las habilidades de los estudiantes para hacer matemáticas.

Palabras clave: Geometría 3D, Diseño de tareas, Competencias del profesor, Enfoque Ontosemiótico.

INTRODUCTION

Didactic of mathematics has consolidated the idea that effective teaching requires, teachers not only master content but also have in-depth didactic knowledge, that allows them to design pedagogical strategies that favor learning (Pino-Fan *et al.*, 2015). From this perspective, one of the central focuses in research has been the characterization of mathematics teachers' professional knowledge and the analysis of strategies for its development.

Several studies have identified the types of knowledge required for teaching (Ball *et al.*, 2008; Beswick & Chapman, 2012; Carrillo-Yañez *et al.*, 2018). Among these, task design occupies a relevant place, as it allows the teacher to act as a mediator between the curriculum, the mathematical content and the needs of their students (Pinto *et al.*, 2022). Designing tasks involves a reflective process about the concepts, the context and the formulation of questions that enhance learning (Watson & Mason, 2006).

It is also recognized that the design of tasks can promote connections between the real world and mathematical concepts, facilitating the transfer of knowledge between contexts (Vargas *et al.*, 2023). Mathematical tasks are therefore fundamental tools for the development of logical thinking and conceptual understanding (Baumert *et al.*, 2017; Watson & Ohtani, 2015). In this process, teachers mobilize didactic-mathematical knowledge and put their creativity to the test to propose meaningful learning situations (Jones & Pepin, 2016; León *et al.*, 2013). However, designing tasks is not a trivial activity, as it requires a complex articulation of knowledge to ensure its potential for the development of knowledge.

Within the framework of task design, the analysis of so-called "eureka moments" or "¡aha!" moments (Liljedahl, 2005), understood as moments of sudden and revealing unders-

tanding during the resolution of a problem, is of special interest. These episodes, linked to creative and intuitive processes (Kounios & Beeman, 2015), offer clues about the suitability and potential of a task to generate significant discoveries. Designing tasks that provide this type of experience requires balancing the level of challenge, promoting reflective pauses and considering non-linear thinking as part of the solution (Danek et al., 2013).

For their part, different studies have approached the study of mathematical tasks from multiple perspectives, focusing on their design, their formative implications and the criteria that define their quality (Watson & Ohtani, 2015; Radmehr, 2023; de Gamboa et al., 2015). However, much of the literature has focused on the final products or on the knowledge gained during implementation, leaving the reflective process that gives rise to the tasks in the background (Watson & Ohtani, 2015). This, in line with the previous comments in this introduction, reinforces the relevance of investigating the experiences, decisions and critical moments that emerge during the very process of designing tasks for mathematics teaching.

In this context, this article seeks to contribute to research into the design of tasks for the teaching of mathematics, based on the formulation of the following research questions:

- **RQ1:** How are teachers' mathematical and didactic knowledge articulated throughout the process of constructing a geometry assessment task?
- **RQ2:** What decisions and strategies emerge as key in the reflective and reasoned design of mathematical tasks?
- **RQ3:** What elements make it possible to structure criteria for evaluating the quality and formative potential of a mathematical task?

To answer these questions, the aim of this study is to analyze the process followed by two teacher-researchers during the design of a task aimed at evaluating geometric objects. This analysis seeks to identify the decisions made, the didactic-mathematical knowledge mobilized and the professional competencies that emerge, with the aim of providing empirical evidence on the process of reflective design—an aspect that is poorly documented in the literature—and to contribute to the construction of criteria that allow the quality of the tasks to be valued, as well as to the generation of proposals that are rich in meaning, challenge and training potential.

THEORETICAL FRAMEWORK

The theoretical framework will cover two main areas. Firstly, an updated overview of the design of tasks for teaching geometry will be presented, based on different perspectives developed in the field of mathematics didactics. Secondly, the Ontosemiotic Approach will be introduced as a conceptual and methodological tool which, on the one hand, allows for an in-depth analysis of the situations described in this research, and on the other, provides a theoretical framework that guides subsequent reflections and discussions.

ASPECTS OF TASK DESIGN

Several principles govern the design of tasks within the framework of the teaching and learning of mathematics. Some authors have synthesized these elements and currently it is possible to recognize that when designing tasks, one should take care of some of the following principles: (a) formulate problems, address mathematical questions and activities and take into account elements of the cognitive demand of the tasks to be designed (e.g., Radmehr, 2023), (b) achieving a learning objective in relation to the proposed task, for example developing competencies and processes such as problem-solving, visualization, conjecture and demonstration (Santa et al., 2018) or argumentation and demonstration and, finally, (c) to achieve some educational objective set by the teacher who designed the task, such as monitoring or evaluating students' thinking and learning while working on the task (Edson et al., 2018) or detecting difficulties and levels of knowledge about some mathematical object that emerge when facing the task in question (Vargas et al., 2023).

In the same vein, mathematics didactics states that learning to design tasks for the development of geometric thinking, in line with current curriculum guidelines, is one of the great challenges in teaching. The selection, creation and adaptation of mathematical tasks is an arduous task that requires extensive knowledge and competence (Mallart et al., 2018). It is also known that both practicing, and in-training teachers strengthen their knowledge, advance in the acquisition of competencies and use mathematical and didactic concepts, properties and procedures that strengthen their teaching work (Guberman & Leikin, 2013). However, despite the consensus on the importance of designing and selecting tasks in mathematics, it is also known that it is difficult to differentiate between routine and non-routine tasks and that choosing appropriate formats to develop students' competencies is not always an easy activity for teachers (Galant, 2013).

To determine whether a task is appropriate or not, Stein & Smith (1998) proposed some elements to consider such as: (1) the level of the students to whom the task is addressed, (2) the knowledge they must be able to complete and (3) the age and abilities of those who solve the task. It is defined that a task will be appropriate if it has the potential for students to develop high-level thinking, and four levels are proposed to classify a task: memorization, procedures without connections, procedures with connections and tasks that involve doing mathematics. Tasks with low cognitive demand are those which seek memorization, or which aim to achieve an answer based on procedures without connections; while tasks with high cognitive demand are defined as those which have some procedure with connections and which, at the end of the day, seek to make the student do mathematics.

Finally, and as an illustration of current research into the design and use of mathematical tasks, Canogullari & Radmehr (2025) propose a review of the literature on this topic, determining four principles that govern current research: Developing problem-solving and

problem-posing competencies, Improving reasoning and proving competencies, Encouraging the use of multiple representations, Encouraging to make connections among mathematical concepts and Cultivating mathematical modelling competencies; within their results they constitute a category referring to the opportunities that tasks give for evaluation, stating that, tasks prepare students to face some test or exam (Leong et al., 2022); they tackle students' misconceptions, mistakes or difficulties (Swan, 2007) and help students to replant solutions and reasoning around the same proposal (Prusak et al., 2012). In summary, it is possible to recognize that tasks are a powerful tool for teaching mathematics; however, they pose major challenges for teachers. Therefore, research that helps to understand and improve the design of tasks as a permanent challenge in teaching is justified and relevant, as it will allow for the constant improvement of a useful tool at all educational levels.

Aspects related to the Ontosemiotic Approach

Research into the didactics of mathematics has focused its attention on identifying and defining the knowledge and competencies that a mathematics teacher must possess to provide effective teaching. As a result of this interest, the Ontosemiotic Approach introduced the mathematics teacher's Didactic-Mathematical Knowledge and Competencies (DMKC model), conceived as an integrative structure that organizes and streamlines both the knowledge and competencies of the mathematics teacher (Pino-Fan & Godino, 2015; Godino et al., 2016). In a nutshell, the DMKC considers that the key competencies of mathematics teachers are mathematical competence and didactic analysis and intervention competence. It is considered that the knowledge required for teaching mathematics implies a thorough knowledge of mathematics and its teaching, i.e. a didactic-mathematical knowledge, since merely mathematical knowledge is not sufficient for adequate practice by mathematics teachers (Pino-Fan et al., 2015).

Knowledge of the mathematics teacher from the DMKC

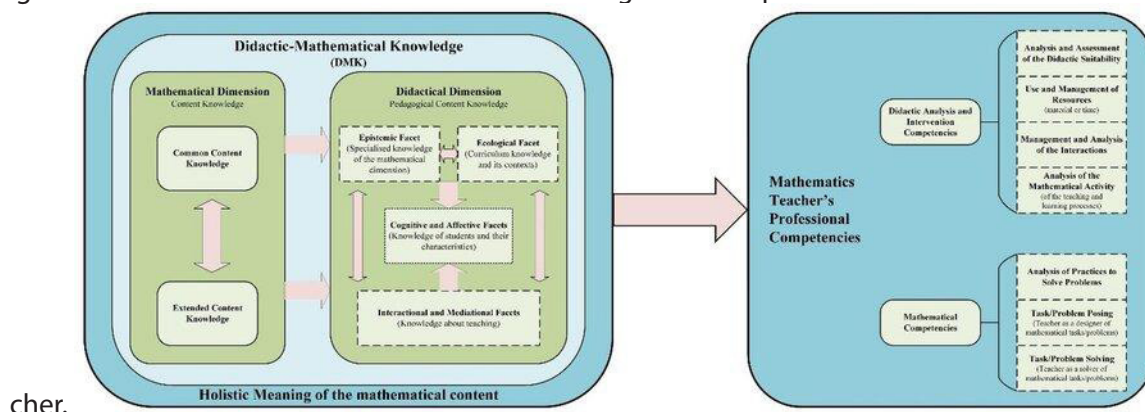
The DMKC model is articulated in three fundamental dimensions (see Figure 1): mathematical, didactic and metadidactic-mathematical. The mathematical dimension distinguishes between two types of knowledge that the teacher must manage (Pino-Fan & Godino, 2015).

Common content knowledge: This refers to the teacher's mastery of a specific mathematical object, enabling them to solve tasks or problems in the curriculum and educational materials at the level they teach. This knowledge is shared between the teacher and the students.

Expanded content knowledge: This implies a deeper understanding of the mathematical object, considering its relevance and connection to content at higher educational le-

vels. This knowledge enables teachers to trace learning paths, design challenging activities, establish links between different mathematical concepts and relate them to both internal and external contexts to the subject.

Figure 1 - Model of didactic-mathematical knowledge and competencies of the mathematics teacher.



Source: Adapted from Pino-Fan et al. (2022, p. 7).

In addition to mathematical knowledge, the DMKC model emphasizes the importance of teachers having a broad understanding of the factors that affect teaching and learning. In this sense, the didactic dimension of the model is broken down into six facets.

- **Epistemic facet:** encompasses specialized mathematical knowledge. In this study, it includes the presence of multiple representations, varied solution strategies rich in mathematical processes, and elements fostering argumentation, clarification, and refinement of definitions and procedures.
- **Cognitive facet:** relates to understanding students' cognitive processes. Here, it concerns whether the task integrates extension activities and builds upon prior knowledge.
- **Interactional facet:** addresses classroom interactions among participants in the teaching-learning process. This study focuses on whether the task promotes group work and opportunities for formalization.
- **Mediating facet:** involves knowledge of resources and means to enhance learning, including time management. In this context, it considers the estimated completion time and use of material resources.
- **Affective facet:** pertains to students' affective and attitudinal dimensions. It includes the task's motivational appeal, its emphasis on the relevance of geometry, and opportunities for student engagement in problem-solving.
- **Ecological facet:** concerns curricular, contextual, social, and economic factors impacting mathematics teaching and learning. The study examines whether the task fosters intra- and extra-mathematical connections, targets an appropriate educational level, and aligns with its intended content and objectives.

Finally, the metadidactic-mathematical dimension of the DMKC model addresses the knowledge needed for teachers to systematically reflect on their practice, evaluate the tea-

ching and learning processes implemented and propose improvements for future interventions (Pino-Fan et al., 2022).

Competencies of the mathematics teacher from the DMKC

The DMKC model identifies two essential competencies for teachers' professional practice: competence in didactic analysis and intervention, and mathematical competence. The first is grounded in the design, implementation, and evaluation of teaching and learning sequences—both personal and external—through didactic analysis techniques and quality criteria, fostering continuous cycles of planning, execution, assessment, and improvement (Font, 2011; Pino-Fan et al., 2015). Its development requires knowledge of both the mathematical dimension (to describe and explain teaching-learning processes) and the meta-didactic-mathematical dimension (to evaluate and enhance these processes) of the DMKC model (Breda et al., 2017). This competence encompasses four sub-competencies (Pino-Fan et al., 2022): analysis and evaluation of teaching competencies, analysis of mathematical activities, use and management of resources, and management and analysis of interactions. Given this article's focus, the first two are detailed:

Sub-competence in the analysis and evaluation of didactic suitability: Based on the concept of didactic suitability (Godino, 2013), this sub-competence enables understanding, development, and reflection on mathematical teaching and learning processes—both one's own and those of others—before, during, and after implementation. It involves six criteria of didactic suitability, each with specific components and indicators aligned with the facets of the DMKC model's didactic dimension (Breda et al., 2017). It supports questions such as: What is the degree of didactic suitability of the proposed geometry task? What improvements could enhance its suitability?

Sub-competence in the analysis of mathematical activity: Involves analyzing both the mathematical practices anticipated by the teacher (a priori analysis) and those demonstrated by students (a posteriori analysis), as well as applying didactic-mathematical knowledge to interpret emerging mathematical activity throughout the instructional process. It addresses questions like: What partial meanings are developed or required in the task? How are these meanings articulated? What configurations of primary mathematical objects and processes underlie these practices? (Font et al., 2013).

Conversely, mathematical competence requires knowledge of the mathematical dimension (for solving mathematical tasks/problems) and the epistemic facet of the didactic dimension (for planning them) within the DMKC model (Pino-Fan et al., 2022). It includes three sub-competencies: analysis of problem-solving practices, planning of tasks/problems, and solving of tasks/problems. As with the previous section, only those sub-competencies most pertinent to this article will be elaborated upon.

Sub-competence in planning tasks/problems: Involves the use of the knowledge of the epistemic (in this specific case to propose tasks for the evaluation of a geometric object), cognitive (given the consideration of possible conflicts or errors of the students in the resolution), and affective (by considering the interests of the students) facets of the DMKC model to thus propose mathematical tasks/problems for the study of a specific notion.

Sub-competence in the resolution of tasks/problems: Involves the resolution that teachers propose, from the perspective of producer/resolver of tasks in mathematics. To develop this sub-competency, teachers must be able to implement the mathematical practices needed to solve mathematical tasks at a particular educational level (such as 2D geometric figure tasks/problems) and, on the other hand, know how to articulate this knowledge with later content (such as the study of 3D geometric bodies) (Godino et al., 2017).

METHODOLOGICAL ASPECTS

In this work we present a qualitative autoethnographic (Adams et al., 2015) case study (Yin, 2009). Its objective is to analyze in depth the process of designing a mathematical task oriented to the evaluation of geometric objects, based on the systematic reflection of their own designers—the authors of this article—who act simultaneously as researchers and participants in the study.

Research Context

The research was organized into three phases: (i) Task design, (ii) Analysis and reflections, and (iii) Conclusion (see Figure 2). The authors of the study acted simultaneously as participants and researchers, playing the dual role of teacher trainers and researchers in Mathematics Didactics. The study originated in an experience of self-reflection following the joint design of a task.

Phase 1: Task design

This stage covered the entire process, from the conception of the idea to the preparation of the final proposal. Specific conditions were set for the design: the task had to be oriented towards the evaluation of a geometric object (3D bodies), include the problem-solving process and present a high level of cognitive demand.

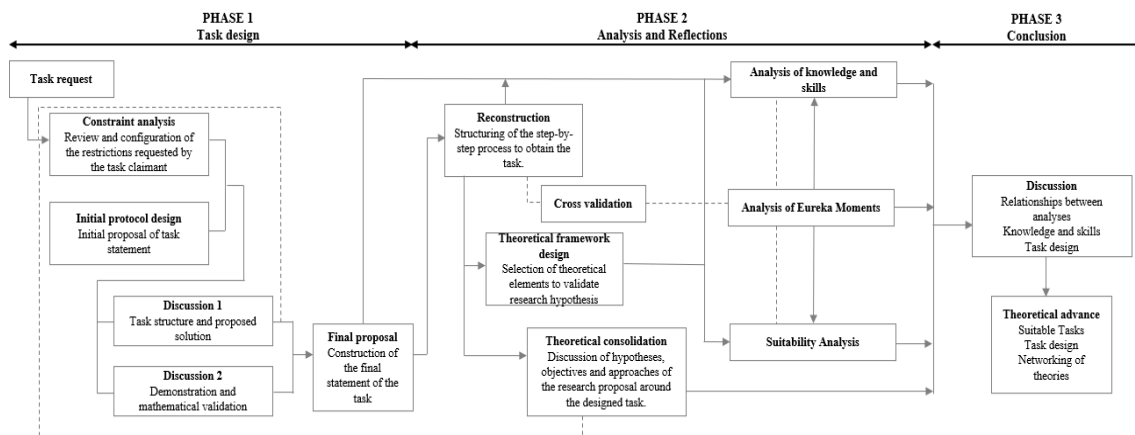
Firstly, an analysis of constraints was carried out, in which the conditions for creating the task were reviewed, which made it possible to define the action parameters of the design. The initial protocol was then designed, focusing on the use of the notion of *surface area* as a trigger for exploring three-dimensional configurations.

Subsequently, two discussion sessions were held. In Discussion 1, one of the researchers presented the proposal to the other, initiating a collaborative process of review and

adjustment. In Discussion 2, we delved into the mathematical validation of the solutions, through calculations and justifications that allowed us to confirm the viability of the proposal. As a result of this phase, the final task statement was defined.

The experience gave the researchers a favorable perception of the quality of the task, which acted as a form of implicit validation of its quality. This recognition gave rise to the need to analyze the process in depth and systematize the decisions taken.

Figure 2 - Methodology



Phase 2: Analysis and reflection

This phase began with an individual reconstruction of the design process. Each researcher prepared a written account detailing their experience, including their reasoning, decisions and reactions to the challenges of design. Subsequently, both texts were shared and contrasted, giving rise to a cross-validation that integrated both perspectives into a single document called the *Path Problem*.

Based on this reconstruction, theoretical references were selected to support the emerging hypotheses about the design of the task. At this stage, a theoretical consolidation took place, in which the researchers debated the conditions of the problem, the restrictions incorporated, the resolution strategies envisaged and the potential didactic quality of the proposal.

On this basis, three complementary analyses were carried out:

1. Analysis of knowledge and competencies, through the application of the mathematics teacher's didactic-mathematical knowledge and competencies model (DMKC), allowing the identification of the knowledge moved during the design.
2. Analysis of Extra-Mathematical moments, in which key moments of sudden comprehension linked to the creative process and validation of task were recognized.
3. Suitability analysis, also from the DMKC model, to evaluate the learning opportunities offered by the task and its didactic limitations.

Phase 3: Conclusion

In this last phase, the results of the previous analyses were integrated, allowing relationships to be established between the different types of emerging knowledge and competencies. At the same time, key contributions were systematized for the discussion on the design of mathematical tasks, especially in relation to the evaluation of geometric objects. The present article is the result of this final reflection and seeks to offer a theoretical and methodological contribution to guide future research and didactic design experiences.

Study Participants

The participants in this research are fundamentally the authors of the article themselves, since the study is based on an activity of self-reflection. This process started with the joint design of the task, followed by a shared discussion of the experiences surrounding it, and culminated in the construction of theoretical and methodological elements derived from this experience. Both researchers are doctors in Mathematics Didactics and have approached various phenomena linked to the teaching and learning of this subject from different theoretical perspectives. One aspect to bear in mind is that both have specific basic training in mathematics, which gives them an in-depth understanding of both mathematical and didactic-pedagogical objects. This double vision is strengthened by their experience in educational contexts at different levels.

Data Collection & Analysis Techniques

To identify the decisions, strategies, knowledge, and competencies emerging during the design of a mathematical task from a reflective and grounded approach, a qualitative analysis of the Path Problem document was conducted. This document systematically reconstructs the process experienced by the researchers in designing and validating the task, resulting from cross-validation of both participants' individual reflections. The Path Problem chronologically records each step, from the initial idea to the final formulation, detailing actions taken, mathematical reasoning, decisions made, discarded alternatives, difficulties encountered, and the justifications underlying each stage. Thus, it served as the primary source for data collection and analysis. The analysis was structured in four main stages:

1. *Initial coding and theoretical categorization:* A qualitative content coding strategy was developed (Neuendorf, 2017), based on a system of predefined categories based on the Didactic-Mathematical Knowledge and Competencies Model (DMKC). The main categories corresponded to the knowledge dimensions of the model, together with the competencies and sub-competencies described in the DMKC. Specific emergent categories were also added to identify the so-cal-

led Eureka moments. The complete set of categories and subcategories used is shown in Table 1, along with the associated codes and descriptors for the coding.

Table 1. Analysis categories

	Analysis category	Subcategory (code)
Knowledge of the mathematics teacher	Mathematical Dimension	Common Knowledge (CCK)
		Extended Knowledge (ECK)
	Didactic Dimension	Cognitive facet (CF), Epistemic facet (EF), Interactive facet (IF), Ecological facet (EcF), Media facet (MF), Affective facet (AF)
Competencies of the mathematics teacher	Competence in didactic analysis and intervention	Sub-competence in the analysis and evaluation of didactic suitability (DIACs1)
		Sub-competence in the analysis of mathematical activity (DIACs2)
	Mathematical competence	Sub-competence in planning problem tasks (MCs1)
		Sub-competence in solving problem tasks (MCs2)

2. *Individual data analysis:* Each researcher independently carried out an analytical reading of the *Path Problem*, applying the categorization system to identify relevant units of meaning. This analysis made it possible to recognize segments of the text where mathematical knowledge, didactic facets, professional competencies (for example, in planning or solving problems), as well as key pedagogical decisions, were evident.
3. *Triangulation and shared coding:* After the individual analysis, a triangulation of the results was carried out: the researchers contrasted their coding, discussed disagreements and consolidated a consensual coding system. This process of cross-validation made it possible to achieve high degree of intersubjective reliability in the interpretation and analysis.
4. *Interpretive analysis with an inductive approach:* At a later stage, the *Path Problem* was revisited from an inductive approach typical of grounded theory (Tepo, 2015), to identify patterns, relationships and emerging elements not initially contemplated in the theoretical categories. *Extra-mathematical* (EM) moments were analyzed, i.e., episodes in which personal experiences, contextual (or non-mathematical) elements or non-formal intuitions had an impact on design decisions (e.g., moments of ‘surprise’ experienced by the researchers and, later, identified as ‘a-ha!’ moments). These moments were contrasted with the predefined analytical categories and discussed in depth during the results interpretation phase. This process made it possible to map with precision the cognitive and professional pathway involved in the task of designing a didactic proposal, as well as to visualize how the different types of knowledge and competencies are articulated in real mathematical and didactic production.

PRESENTATION AND ANALYSIS OF RESULTS

In this section we present and analyze the results obtained. The first section contains the researchers' reconstruction of the task and its initial design. Subsequently, there is a subsection on the professional knowledge of the mathematics teacher detected and analyzed, which leads to a third section in which the competencies highlighted are discussed. Finally, you will find some elements relating to the extra-mathematical moment experienced.

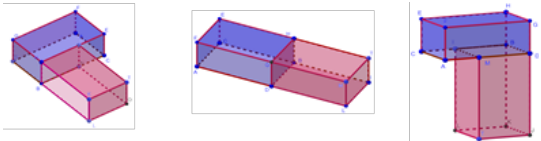
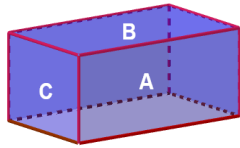
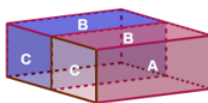
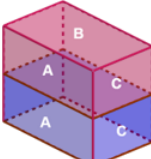
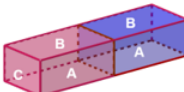
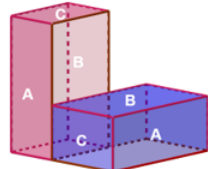
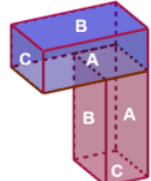
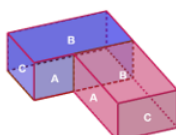
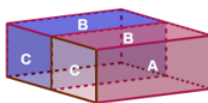
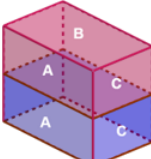
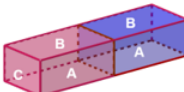
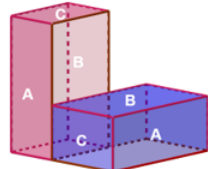
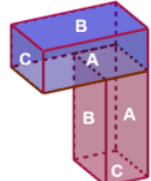
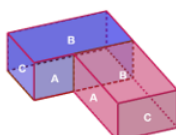
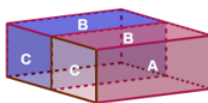
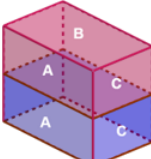
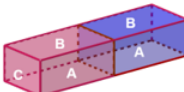
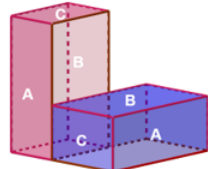
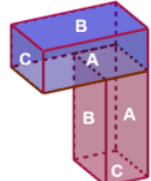
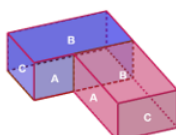
Path Problem

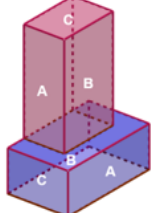
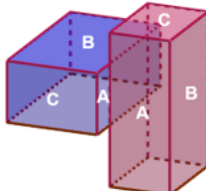
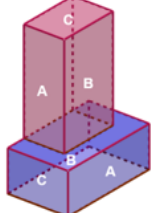
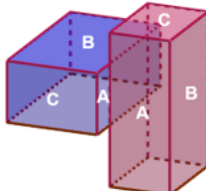
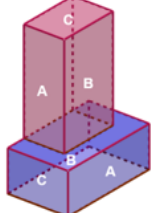
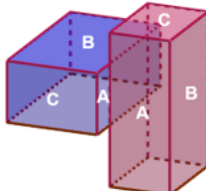
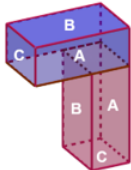
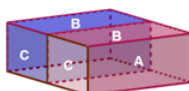
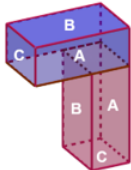
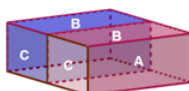
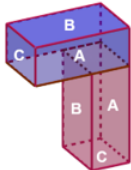
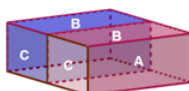
Table 2 shows the reconstruction of the step-by-step process followed to achieve the final task. It details the discussions, proposals and elements that led to the final statement of the task implemented, as well as some elements that emerged during this process. Each section has been separated (according to what were identified as subsequent steps of advance in the design and analysis of the task) to make it easier to read and to classify it into one of the categories shown in Table 1.

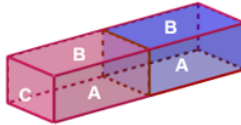
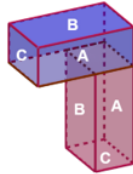
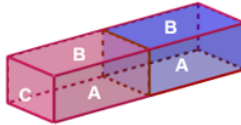
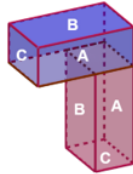
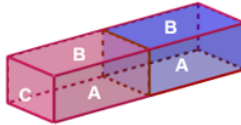
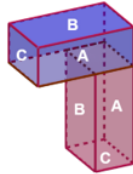
Table 2 - Path Problem

Item	Analysis unit	C-S3
1	The aim is to design a task for the assessment of certain school geometry competencies. The aim is to create a problem related to bodies and figures in 3D, whose solution leads middle school students to put into play different processes, tools and knowledge of geometry, in such a way that it is possible to evaluate the achievement or not of these types of knowledge.	MCs1
2	It is based on the idea that the configuration of groups of bodies in 3D is not a usual activity in the study of geometry, which is why activities involving the visualization, idealization, configuration and construction of structures in 3D could add richness to the process of solving a problem. In this sense, a well-known figure is chosen, from which the structure of the problem begins. The researchers' initial reference point during the task design process was evoked by the toys they had been familiar with since childhood, particularly the construction blocks they used to assemble, such as Lego.	CF EcF DIACs2 ECK EF EM
3	The body selected is a rectangular prism. To make the problem more general and not become an arithmetic calculation activity, the figure will be given in a standard position, without established measurements and without the possibility of determining them.	CCK EF MCs1 DIACs2 ECK
4	For this process, an idea emerges spontaneously: depending on how you put together the two blocks, the measure of the total surface varies and then a question emerges: In which configuration do I, then, get the maximum surface?	IF MCs1
5	Now, the problem that you want to ask consists of the following statement: <i>“When you take two geometric bodies like the ones shown in the image, which configuration has the largest possible total surface area?”</i> Where together with the task's statement, an example of the prisms is provided.	MCs1

³ C8-S: Categories-subcategories, the codes used in this column are specified in Table 1

6	Initially, the idea was for the student to be able to structure configurations like the ones described below and then check the surface area of each one, so that they could achieve the configuration with the largest area. 	DIACs2 EcF												
7	Initially we tried to look for the possible configurations that existed in the structure of the figure, but it was necessary to set some elements to control the variability and the multiple response options that were appearing. This part is perceived as an obstacle that could invalidate the idea of the task. This observation is followed by a moment of silent reflection. In this sense, we decided to establish an order between the faces of the figure, so that comparisons could be generated. The figure corresponds to a rectangular prism whose surfaces show that $A > B > C$ 	CCK DIACs2 DIACs1 MCs1 EcF												
8	At this stage, the feeling is that the task is solvable without having to go into too much detail or introduce overly complicated reasoning. This is the first moment when a constraint is introduced.	EF DIACs2												
9	<i>“If you take two rectangular prisms like the ones shown in the picture and you fulfill that $A > B > C$, which configuration has the largest possible total surface area?”</i>	MCs1												
10	From this consideration, two options emerge: (1) implement a multiple-choice task; (2) implement a constraint in the configuration. Choosing option (1) would have narrowed down the reasoning behind the different configurations quite a bit, since they would be proposed directly as options. It remains, however, a valid option with the aim of simplifying the task a little. On the other hand, option 2 makes the problem more complex in the sense that it allows a broader and deeper exploration of the situation described.	ECK MCs1 EF												
11	Now that an orderly relationship has been established between the surfaces of the figure, it has been possible to find all the possible configurations (The initial configurations are shown, and it is accepted that it is possible to generate some kind of variation associated with the relative position of both prisms). <table border="1" data-bbox="379 1370 1165 1818"> <thead> <tr> <th>AA</th><th>BB</th><th>CC</th></tr> </thead> <tbody> <tr> <td></td><td></td><td></td></tr> <tr> <th>AB</th><th>BC</th><th>AC</th></tr> <tr> <td></td><td></td><td></td></tr> </tbody> </table>	AA	BB	CC				AB	BC	AC				EF ECK FM
AA	BB	CC												
														
AB	BC	AC												
														

12	The analysis of the possible configurations also included some cases, the first being an invalid case that had to be controlled by the statement, so it is not possible to solve the problem without controlling that the entire contact face must actually be in contact with the entire face of the other prism. To illustrate this, the following image is presented. <table><tr><th>POSSIBLE CASE</th><th>CASE NOT POSSIBLE</th></tr><tr><td></td><td></td></tr></table> The possible case corresponds to that in which the contact faces have a known relationship, in the configuration shown in the table it corresponds to B-C. On the other hand, the case that is not possible is represented by the right-hand side of the table, so it is not possible to calculate the resulting value without the contact faces which try to establish a relationship between B and A, but which, given the position of the prisms, is not possible to calculate.	POSSIBLE CASE	CASE NOT POSSIBLE			DIACs1 EF FM					
POSSIBLE CASE	CASE NOT POSSIBLE										
											
13	The problem now needed to be organized according to the total surface area and consider some additional elements so that the statement did not allow any additional variation to those established and considered in table 2. In this way, the wording of the question would now be as follows: “By taking two rectangular prisms like the ones shown in the image and in which we fulfill that $A > B > C$, putting the prisms together in such a way that their Surface of contact consists in a whole phase of one of the two prisms. What is the configuration of the largest possible total surface area?”	CCK MCs1									
14	Consideration: configurations must meet one of the following two conditions: • If the prisms are joined by a face of the same denomination (A, B or C), both faces must be completely joined. • If the prisms are joined by faces of different denominations, one of the contact faces must be completely in contact with the other. Now that the statement has been completely defined, it is necessary to determine, under the given conditions, which configuration has the largest surface area to be able to verify the possible solution (s) and also validate the task from a mathematical point of view.	DIACs1 DIACs2									
15	The following table shows two examples of surface areas associated with their initial configuration. The remaining cases are similar, observing that the “mixed” configurations (AB, AC, BC) have the same surface area value; as do the “elementary” configurations (AA, BB, CC). <table><tr><th>CASE</th><th>INITIAL CONFIGURATION</th><th>AREA</th></tr><tr><td>CASE BC</td><td></td><td>$2A + 2C + B + (B - C) + 2A + 2B + C$$= 4A + 4B + 2C$</td></tr><tr><td>CASE AA</td><td></td><td>$A + 2C + 2B + A + 2B + 2C$$= 2A + 4B + 4C$</td></tr></table>	CASE	INITIAL CONFIGURATION	AREA	CASE BC		$2A + 2C + B + (B - C) + 2A + 2B + C$ $= 4A + 4B + 2C$	CASE AA		$A + 2C + 2B + A + 2B + 2C$ $= 2A + 4B + 4C$	DIACs2 MCs2
CASE	INITIAL CONFIGURATION	AREA									
CASE BC		$2A + 2C + B + (B - C) + 2A + 2B + C$ $= 4A + 4B + 2C$									
CASE AA		$A + 2C + 2B + A + 2B + 2C$ $= 2A + 4B + 4C$									

16	Therefore, all possible cases can be reduced to one of the following three expressions $4A+2B+4C \quad (1)$ $4A+4B+2C \quad (2)$ $2A+4B+4C \quad (3)$ At this moment a moment of surprise was felt as, a-priori, we would have not imagined that the three cases in which the contact faces were not of the same kind (i.e., A with B, A with C, B with C) may be reconducted to the elementary cases of contact faces of the same kind (i.e., A with A, B with B and C with C). At this point, it was necessary to determine the relation between the expressions (1), (2) and (3), to identify the largest one and, therefore, which are the requested configurations. We start by considering the condition $A>B>C$. Let's consider the relation $A>C$. By algebraic manipulation of the expression, we obtain:$2A-A>2C-C$ and$2A+C>A+2C$ and$4A+2C>2A+4C$ and, finally,$4A+4B+2C>2A+4B+4C$, that is .(2)>(3) Similarly, we obtain(2)>(1). With similar reasoning as the one presented above, we conclude that(1)>(3) and, thus, .(2)>(1)>(3) At this point another moment of surprise emerged, by realizing how important and deeply embedded in the solving process the introduced constraint is.	MCs2 EM				
17	This implies that there are two configurations that have the largest possible surface area, by joining two prisms such as those given by the problem; since they are equivalent, any of the two configurations is a solution to the problem <table><tr><th>SOLUTION 1</th><th>SOLUTION 2</th></tr><tr><td></td><td></td></tr></table>	SOLUTION 1	SOLUTION 2			MCs2
SOLUTION 1	SOLUTION 2					
						

Teacher knowledge and competencies

According to the MDKC model, there is ample evidence of both common and extended knowledge in relation to the mathematical knowledge category. The task addresses the idea of surface area on three-dimensional bodies, so the appearance of various extended knowledge linked to 3D geometry was expected. Item 3 shows how, at first, a familiar body (a rectangular prism) is used to start thinking about the activity. This common knowledge allows connections to be made with different extended knowledge. Particularly noteworthy is the ECK present in item 11, where the different configurations of the selected bodies demonstrate a deep knowledge on the part of the researchers, by recognizing unconventional positions and non-prototypical configurations of three-dimensional figures.

In this same vein, the statement of mathematical restrictions, such as the order relationship between the faces of the prism, as well as the design of conditions of exclusivity, sequentiality and configuration of three-dimensional figures (items 7, 9 and 13), constitute additional evidence of mobilized mathematical knowledge, as well as the specialized training of researchers in the field of geometry in 2 and 3 dimensions. Common knowledge is

understood as those elements associated with the fundamental properties of three-dimensional bodies—in this case, the prism—such as surface area, dimensionality and implicit measurements in relation to an ordered structure, as exemplified by the inequality mentioned in item 9. On the other hand, extended knowledge is that which goes beyond the expected level, involving more complex or non-prototypical reasoning; in this sense, the construction of configurations from two figures, the introduction of restrictions such as those in item 12, and the mathematical analysis developed in item 16, are clear manifestations of this type of knowledge. The researchers' expertise is evident in their ability to justify, from a rigidly mathematical perspective, a property that underpins the proposed task, as well as in their ability to advance towards the formulation of a final statement that coherently integrates the totality of the constraints and knowledge involved.

In relation to the knowledge typical of the didactic dimension and the facets that allow us to identify the type of knowledge that emerges, it is pertinent to highlight the marked emphasis on the *epistemic facet* (EF). This predominance is understandable, given that the task has an eminently mathematical approach which, although it seeks to generate a connection with an element of the everyday environment—evidence of the *ecological facet* (EcF), as can be seen in item 2—, requires in-depth knowledge of geometric objects and their properties, as well as arithmetic and algebraic concepts, such as the relationships of order and inequality. Items 2, 3, 8, 10, 11 and 12 provide significant evidence of this type of knowledge, by highlighting the emergence of different solution paths, the richness of the cognitive processes involved and the revision of definitions and procedures that formed part of the task's design.

The *cognitive facet* (CF) is approached in the same way, but to a lesser extent. In this case, the knowledge of the students to whom the task would be addressed is particularly important, as the task aims to lead the student down a path of solution in which their previous knowledge plays a decisive role (see items 3 and 8); in this sense, to be able to solve the task, students are expected to use their knowledge about the area of 2D figures, particularly polygons, making use of these ideas, they should be able to access new knowledge about 3D. Knowing that these elements are previous concepts required to solve the proposed task is considered to be evidence of knowledge around the cognitive facet.

The *ecological* (EcF) and *mediational facets* (MF) constitute the next dimensions identified in the analysis. Firstly, items 2, 6 and 7 show the presence of the *ecological facet*, as they show the use of different representations of the same figure or three-dimensional body, which can be interpreted as an intra-mathematical connection of the *representation* type. In addition, the reference to the use of a figure known to the students, which evokes a playful object—specifically, a game—is a manifestation of an extra-mathematical connection, which reinforces the relevance of this facet in the design of the task.

On the other hand, items 6, 11 and 12 constitute relevant evidence of the *mediational facet*. Despite the absence of concrete physical structures (such as rectangular prisms), there is a clear knowledge of teaching resources and media, expressed through the idealization of these objects. This ability to conceptualize and use mental representations of three-dimensional bodies shows how geometric knowledge is activated when it comes to solving the task, making it possible to explore different configurations and spatial positions depending on the proposed objective.

Although the *affective facet (AF)* is not formally explained in this section, the fact that it incorporates elements linked to the student's personal experience can be interpreted as attention to their interests and motivations. In this sense, a manifestation of this facet could be recognized—albeit to a lesser extent—by considering how the emotional bond and the recognition of known referents can favor commitment to the task.

Finally, it is worth noting that the *interactional (IF)* and *affective facets (AF)* were not explicitly expressed during the development of the activity. This absence was predictable, given that the task was not designed to foster interactions between students, nor between them and the teacher, but was designed to be completed individually, to assess knowledge of a specific geometric object. In this same sense, the lack of evidence corresponding to the *affective facet* can be understood from the evaluative focus of the task, centered on the individual achievement of mathematical knowledge, rather than on the consideration of motivational or emotional factors in the solving process.

In line with the above, it is possible to establish a direct connection between the didactic-mathematical knowledge demonstrated by the researchers and the presence of some of the professional competencies defined in the MDKC model. Relevant manifestations of *competence* are identified in the *analysis and didactic intervention*. A first noteworthy aspect is the identification of elements typical of *sub-competence in the analysis of mathematical activities* (DIACs2). During the design process, the researchers showed systematic attention to a priori analysis, based on their knowledge of task design and geometry in two and three dimensions. This analysis made it possible to anticipate possible answers and solution paths that could be followed by the students when approaching the task. For example, items 2, 3, 6 and 7 reveal how, when reflecting on these possible solutions, the need to declare restrictions linked to the measurements of the faces of the prisms emerged, as well as the convenience of establishing specific positions to avoid uncontrolled configurations, such as those observed in item 12.

In this same vein, the researchers showed a deep knowledge of mathematical objects and processes linked to 3D geometry, as well as partial meanings that can arise during mathematical activity. This is reflected in items 8, 14 and 15, where it is acknowledged that, as a result of their experience in teaching this area, there are definitions and procedures

that are often not covered in depth, but which are essential for properly assessing geometric knowledge. Examples of this are the use of figures in unusual positions or the conceptualization of surface area as the composition of multiple base structures.

A second relevant aspect is the identification of *sub-competency* elements in the *analysis and evaluation of didactic suitability* (DIACs1). In this sense, the researchers showed a sustained reflective capacity regarding their practice, both in the previous phases and during and after the design of the task. The explicit identification of four of the six facets of the model, as well as the integration of elements from each of them in the final task, is evidence that didactic suitability was a key criterion in the design process. This approach not only considered the mathematical knowledge involved, but also aspects such as the students' interests, the link between the knowledge and everyday experience, possible solution paths, and the prior knowledge needed to achieve the proposed objectives.

Regarding the *mathematical competence* of the MDKC model, significant contributions have also been identified, particularly in relation to the *sub-competence of planning and solving tasks*. Firstly, the researchers showed an interest in designing a task aimed at evaluating a specific geometric object, using an unconventional structure within the usual teaching of geometry, as can be seen in items 2, 3 and 4. This approach led them to solve the task themselves, in order to anticipate possible difficulties that might arise during its implementation, as well as the questions that the students might ask. Items 4, 5, 9, 10 and 13 illustrate this reflective journey, where the potential challenges involved in solving the task emerged. The decisions adopted, such as the design of restrictions expressed through inequalities or explicit statements, show a solid competence in the design and formulation of structured tasks, in which the student's transit is not completely free, but oriented towards the achievement of the mathematical knowledge envisaged.

Finally, items 15, 16 and 17 demonstrate the researchers' competence in solving tasks and problems. In these cases, it is evident that they have a profound mastery of 3D geometry, by directing the task towards the object of evaluation, articulating concepts from geometry in 2 and 3 dimensions, and justifying their decisions from both a mathematical and didactic perspective. Thus, the process of solving the task integrates a view centered on the knowledge of the content, but also on the student's experience and the teacher's practice. Overall, the design of the task is a concrete manifestation of competence in planning and solving tasks or problems and reflects the strength of both the didactic and mathematical knowledge of those responsible for its preparation.

Extra-mathematical moments

In the analysis of the Path Problem, three extra-mathematical moments were identified (items 2, 7 and 16). These moments, perceived as significant by both researchers, oc-

curred at very different stages of the task design process and in different contexts. In the first case (item 2), there was an instance in which ideas from the authors' personal experiences were incorporated—such as the use of games, specifically building blocks—which provided a significant basis and an initial direction on which the task was structured.

Although the task does not explicitly refer to real-world objects, its origin has a potential connection with the non-mathematical environment, which can become a point of meaning for the students when approaching the activity. In general terms, from the perspective of the task design, this moment was positively valued for its contribution to making the process of generating a didactic situation visible. Although item 7 highlighted a moment of silent reflection as highly significant for the design of the task, this reflection emerged from parallel processes of individual analysis by the researchers of the situation posed. In this sense, it is relevant to emphasize the need for continuous reflection on the mathematical objects involved in the design of a task, even though moments of apparent disconnection with the activity itself, allowing a space for the development and maturation of mathematical thinking without direct intervention on the task.

A third extra-mathematical moment occurred in item 16, during the process of resolution of the task, in the validation phase. It was observed that, at this stage, the researchers were still exploring whether the “non-standard” cases could be tackled easily or whether it was necessary to introduce new restrictions to the formulation of the task. For this, algebraic analysis of the expressions representing the total surfaces was used.

As the calculations were carried out and the data from the statement was incorporated, all the algebraic expressions were simplified until they became a manageable set of expressions that facilitated access to the problem's solution. This result was perceived as surprising, since there were no previous indications that the solution would be so accessible. Faced with this finding, both researchers agreed that, at this point in the development, the task revealed its full potential, generating a sensation of “liberating surprise”, which a-posteriori was identified as a moment of revelation or “a-ha! moment”. This moment, full of intuition and discovery, functioned as an implicit validation of the task's quality: after trying it out, it was unanimously concluded that the designed task was a “good task” (according to the spontaneous and natural words that emerged after this moment).

From this perspective, it is interesting to observe how the entire process of designing the task responded to a construction “from the bottom up”, that is, without relying on a solution foreseen in advance. This characteristic, in principle, could represent a difficulty in guaranteeing the suitability of the task, since there is a risk that the designer will propose a task that exceeds the students' mathematical abilities (for example, if you consider a generalized version of the problem in which two blocks are combined without specific restrictions, the solving process changes radically and becomes considerably more complex). For this rea-

son, it is important to identify moments in the process when explicit or implicit validation of the suitability of the designed task is achieved.

| DISCUSSION

The results obtained make it possible to provide sustained answers to the research questions posed. Regarding *RQ1: How are teachers' mathematical and didactic knowledge articulated throughout the process of constructing a task for geometry assessment?* it is evident that task design is a powerful tool for favoring this articulation. In agreement with Guberman and Leikin (2013), the design process made it possible to integrate different types of knowledge, while strengthening competencies such as planning and solving problems aimed at teaching mathematics.

As authors and designers, we believe that the very activity of designing a task that guides students through different types of knowledge—including those that are unknown or held back—enables a natural articulation between mathematical and didactic knowledge. In this vein, we share what Jones & Pepin (2016) have said, in that the design process involved activating complex knowledge and putting creativity into play to generate a motivating and, at the same time, robust task from a mathematical point of view.

In relation to *RQ2: What decisions and strategies emerge as key in the reflective and reasoned design of mathematical tasks?* Multiple moments of reflection, reformulation and continuous questioning were identified. Each decision in the design responded to the need to activate specific knowledge on the part of the research team, which showed an iterative and informed analysis process. The time set aside for reflection, research and reviewing the proposal was key to providing moments of discovery, some of which were experienced as authentic *a-ha moments*. According to Liljedahl (2005), these episodes are linked to cognitive experiences in which the solution to a problem emerges suddenly, intuitively and significantly, which was interpreted as an indicator of the quality of the task designed.

Regarding *RQ3: What elements make it possible to structure criteria for evaluating the quality and formative potential of a mathematical task?* Multiple dimensions are recognized as quality criteria. We highlight the value of the Ontosemiotic Approach and the MDKC model as integrating elements that allow us to evaluate the cognitive, epistemic and didactic richness of mathematical tasks. Following Pino-Fan & Godino (2015), we believe that structuring and analyzing teachers' knowledge and competencies not only contributes to improving teaching practice but also enriches understanding of the processes involved in designing tasks with high training value.

In the specific case of the designed task, it was particularly revealing that configurations initially perceived as more complex ultimately related to previously analyzed cases,

prompting a continuous review of the knowledge involved and the preparation required to address emerging challenges. This iterative process led to findings on emerging knowledge and competencies that, while partly unexpected, were interpreted as indicators of the design's maturity, as the task fostered mathematical reasoning, confirmed its value as a didactic proposal, and demonstrated clear potential for implementation in teacher education contexts. In this regard, the articulated design process can serve as a reflective practice within initial and in-service teacher training, supporting the development of didactic-mathematical competencies. Engaging future teachers in the guided design and analysis of mathematical tasks—such as the one presented—can strengthen their ability to integrate mathematical and didactic knowledge, enhance reflective decision-making, and promote the creation of high-quality, cognitively challenging learning experiences. In line with Watson & Ohtani (2015), Pino-Fan & Godino (2015), and Pinto et al. (2022), these findings underscore that the richness of a task lies in the breadth and depth of the knowledge mobilized in its design and resolution, and that systematic use of task design can meaningfully contribute to preparing teachers capable of fostering effective mathematical learning environments.

Finally, it is possible to state that the objective of this research has been achieved, since it has been possible to identify and analyze in depth the decisions made, the didactic-mathematical knowledge mobilized and the professional competencies that emerged during the process of designing a task aimed at evaluating geometric objects. The study contributes to the development of criteria to value the quality of the design of mathematical tasks and provides guidelines for the generation of proposals rich in meaning, cognitive challenge and formative potential, including moments of discovery and sudden understanding. As a research project, the need to explore how the structure and complexity of the task would vary if special cases were incorporated—for example, configurations with unconventional lengths that avoid obvious overlaps—which would open new routes to resolution, broadening the field of analysis and mobilizing other types of knowledge that have not yet been explored.

REFERENCES

- Adams, T. E., Holman Jones, S. & Ellis, C. (2015). *Autoethnography*. Oxford University Press.
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher Education*, 59(5), 389-407. <https://doi.org/10.1177/0022487108324554>
- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M., & Tsai, Y.-M. (2017). Teachers' Mathematical Knowledge, Cognitive Activation in the Classroom, and Student Progress. *American Educational Research Journal*, 47(1), 133-180. <https://doi.org/10.3102/0002831209345157>

- Beswick, K., & Chapman, O. (2012). Knowledge for teaching mathematics. En G. Hanna & M. de Villiers (Eds.), *Proof and proving in mathematics education: The 19th ICMI Study* (pp. 95–118). Springer. https://doi.org/10.1007/978-94-007-2129-6_6
- Breda, A., Pino-Fan, L. R., & Font, V. (2017). Meta Didactic-Mathematical Knowledge of Teachers: Criteria for The Reflection and Assessment on Teaching Practice. *Eurasia Journal of Mathematics, Science and Technology Education*, 13(6), 1893-1918. <https://doi.org/10.12973/eurasia.2017.01207a>
- Canogullari, A., & Radmehr, F. (2025). Task design principles in mathematics education: a literature review. *International Journal of Mathematical Education in Science and Technology*, 1–33.
- Carrillo-Yañez, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., ..., & Muñoz-Catalán, M. C. (2018). The mathematics teacher's specialised knowledge (MTSK) model. *Research in Mathematics Education*, 20(3), 236–253. <https://doi.org/10.1080/14794802.2018.1479981>
- de Gamboa, G., Badillo, E., Ribeiro, M., Montes, M., & Sánchez-Matamoros, G. (2015). Teacher's knowledge and the use of connections in the classroom. *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education (CERME9)*.
- Danek, A. H., Fraps, T., von Müller, A., Grothe, B., & Öllinger, M. (2013). Aha! experiences leave a mark: Facilitated recall of insight solutions. *Psychological Research*, 77(5), 659-669.
- Edson, A. J., Phillips, E. D., Slinger-Grant, Y., & Stewart, J. (2018). The Arc of Learning framework: An ergonomic resource for design and enactment of problem-based curriculum. *International Journal of Educational Research*, 93, 91–108. <https://doi.org/10.1016/j.ijer.2018.09.020>
- Font, V. (2011). Competencias profesionales en la formación inicial de profesores de matemáticas de secundaria. *Unión*, 26, 9-25.
- Font, V., Godino, J.D. & Gallardo, J. (2013). The emergence of objects from mathematical practices. *Educ Stud Math* 82, 97–124. <https://doi.org/10.1007/s10649-012-9411-0>
- Galant, J. (2013). Selecting and sequencing mathematics tasks: Seeking mathematical knowledge for teaching. *Perspectives in Education*, 31(3), 34–48.
- Godino, J. D. (2013). Indicadores de idoneidad didáctica de procesos de enseñanza y aprendizaje de las matemáticas. *Cuadernos de Investigación y Formación en Educación Matemática*, 11, 111-132.
- Godino, J. D., Batanero, C., Font, V. y Giacomone, B. (2016). Articulando conocimientos y competencias del profesor de matemáticas: el modelo CCDM. En J. A. Macías, A. Jiménez, J. L. González, M. T. Sánchez, P. Hernández, C. Fernández, F. J. Ruiz, T. Fernández y A. Berciano (Eds.), *Investigación en Educación Matemática XX* (pp. 285-294). Málaga: SEIEM.

Godino, J. D., Giacomone, B., Batanero, C. & Font, V. (2017). Enfoque Ontosemiótico de los conocimientos y Competencias del Profesor de Matemáticas. *Bolema*, 31(57).

Guberman, R. & Leikin, R. (2013). Interest and difficulty: Changes in teachers' views of multiple solution tasks. *Journal of Mathematics Teacher Education*, 16(1), 33-56.

Jones, K. & Pepin, B. (2016). Research on mathematics teachers as partners in task design. *J Math Teacher Educ* 19, 105–121 . <https://doi.org/10.1007/s10857-016-9345-z>

Kounios, J., & Beeman, M. (2015). The eureka factor: Aha moments, creative insight, and the brain. *Scientific American Mind*, 26(2), 24-31.

León, O. L., Díaz Celis, F., & Guilombo, M. (2013). Diseños didácticos y trayectorias de aprendizaje de la geometría de estudiantes sordos, en los primeros grados de escolaridad. *Revista Latinoamericana de Etnomatemática*, 7(2), 9–28.

Leong, Y. H., Cheng, L. P., Lim-Teo, S. K., Toh, T. L., & Kaur, B. (2022). Instructional materials as a site to study teachers' planning and learning. *Mathematics Education Research Journal*, 34, 7–26. <https://doi.org/10.1007/s13394-021-00368-9>

Liljedahl, P. G. (2005). Mathematical discovery and affect: the effect of AHA! experiences on undergraduate mathematics students. *International Journal of Mathematical Education in Science and Technology*, 36(2–3), 219–234. <https://doi.org/10.1080/00207390412331316997>

Mallart, A., Font, V., & Diez, J. (2018). Case Study on Mathematics Pre-service Teachers' Difficulties in Problem Posing. *Eurasia Journal of Mathematics, Science and Technology Education*, 14(4), 1465-1481. <https://doi.org/10.29333/ejmste/83682>

Neuendorf, K. A. (2017). *The content analysis guidebook* (2nd ed.). SAGE.

Pino-Fan, L. R., Assis, A., & Castro, W. F. (2015). Towards a Methodology for the Characterization of Teachers' Didactic-Mathematical Knowledge. *Eurasia Journal of Mathematics, Science and Technology Education*, 11(6), 1429-1456. <https://doi.org/10.12973/eurasia.2015.1403a>

Pino-Fan, L. R., Assis, A., & Castro, W. F. (2022). University teachers' didactic-mathematical knowledge for teaching the effect of coefficient b on the quadratic function. *International Journal of Mathematical Education in Science and Technology*, Advance online publication. <https://www.researchgate.net/publication/375380272>

Pino-Fan, L., & Godino, J. D. (2015). Perspectiva ampliada del conocimiento didáctico-matemático del profesor. *Revista Paradigma*, 36(1), 87–109.

Pinto, M., Badillo, E., Alsina, Á., & López, A. (2022). El diseño de tareas como competencia profesional del profesorado de matemáticas: un estudio exploratorio con futuros maestros de Educación Primaria. *Revista de Educación Matemática*, 37(3), 5–27. <https://doi.org/10.24844/EM3703.01>

Prusak, N., Hershkowitz, R. & Schwarz, B.B. (2012). From visual reasoning to logical necessity through argumentative design. *Educ Stud Math* 79, 19–40. <https://doi.org/10.1007/s10649->

011-9335-0

Radmehr, F. (2023). Toward a theoretical framework for task design in mathematics education. *Journal on Mathematics Education*, 14(2), 189–204. <https://doi.org/10.22342/jme.v14i2.pp189-204>

Santa, A., Hernández, R., & González, M. J. (2018). La formación del profesorado en el diseño de tareas matemáticas: Un estudio de caso. *Revista Latinoamericana de Investigación en Matemática Educativa*, 21(1), 37–64. <https://doi.org/10.12802/relime.18.2112>

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School*, 3(4), 268–275.

Swan, M. (2007). The impact of task-based professional development on teachers' practices and beliefs: a design research study. *J Math Teacher Educ*, 10, 217–237. <https://doi.org/10.1007/s10857-007-9038-8>

Teppo, A.R. (2015). Grounded Theory Methods. In: Bikner-Ahsbahr, A., Knipping, C., Presmeg, N. (eds) *Approaches to Qualitative Research in Mathematics Education*. Springer, Dordrecht. https://doi.org/10.1007/978-94-017-9181-6_1

Vargas Herrera, J. P., Giménez, J., & Vanegas, Y. (2023). Análisis de conocimientos didáctico-matemáticos sobre clasificación de poliedros con futuros maestros de educación primaria. *Paradigma*, 44(4), 293–320. <https://doi.org/10.37618/PARADIGMA.1011-2251.2023.p293-320.id1404>

Watson, A. & Mason, J. (2006) Seeing an Exercise as a Single Mathematical Object: Using Variation to Structure Sense-Making. *Mathematical Thinking and Learning*, 8(2) p. 91-111.

Watson, A., & Ohtani, M. (Eds.). (2015). *Task Design in Mathematics Education: An ICMI Study 22*. Springer. <https://doi.org/10.1007/978-3-319-09629-2>

Yin, R. K. (2009). *Case Study Research*. SAGE.

Apéndice—Información sobre el artículo

Historico editorial

Submetido: 26 de Junio de 2025.

Aprobado: 16 de Diciembre de 2025.

Publicado: 18 de Enero de 2026.

Como Citar — APA

Vargas, J. P., & Lamanna, L. (2026). "When you find the right task, you'll know": design and validation of a geometric task - a case study. *PARADIGMA*, XLVII(1), e2026008. <https://doi.org/10.37618/PARADIGMA.1011-2251.2025.e2026008.id1779>.

Como Citar — ABNT

VARGAS, Juan Pablo; LAMANNA, Luca. "When you find the right task, you'll know": design and validation of a geometric task - a case study. *PARADIGMA*, Maracay, v. XLVII, n. 1, e2026008, Ene./Jun., 2026. <https://doi.org/10.37618/PARADIGMA.1011-2251.2026.e2026008.id1779>.

Conflicto de intereses

Nada que declarar.

Declaración de disponibilidad de datos

Todos los datos han sido presentados/generados en este artículo.

Derechos autorales

Los derechos de autor pertenecen a los autores, que conceden a revista **Paradigma** los derechos exclusivos de primera publicación. Los autores no serán remunerados por la publicación de sus artículos en esta revista. Los autores están autorizados a celebrar contratos adicionales por separado, para la distribución no exclusiva de la versión del artículo publicado en esta revista (por ejemplo, publicación en un repositorio institucional, en un sitio web personal, publicación de una traducción o como capítulo de un libro), con reconocimiento de autoría y primera publicación en esta revista. Los editores de la revista **Paradigma** tienen derecho a realizar ajustes textuales y adecuación normativa en este artículo.

Acceso libre

Este artículo es de acceso abierto (**Open Access**) y sin gastos de envío ni de procesamiento del artículo (**Article Processing Charges - APCs**). El acceso abierto es un amplio movimiento internacional que pretende proporcionar acceso en línea libre y gratuito a la información académica, como publicaciones y datos. Una publicación se define como de acceso abierto cuando no existen barreras financieras, legales o técnicas para acceder a ella; en otras palabras, cuando cualquiera puede leerla, descargarla, copiarla, distribuirla, imprimirla, investigarla o utilizarla en la educación o de cualquier otra forma dentro de los acuerdos legales.



Licencia de uso

Este artículo es licenciado con **Creative Commons Atribución-NoComercial-SinDerivadas 4.0 Internacional (CC BY-NC-ND 4.0)**. Esta licencia le permite compartir, copiar y redistribuir el artículo en cualquier medio o formato. La licencia no permite utilizar el material con fines comerciales ni adaptarlo, remezclarlo o transformarlo.



Comprobación de similitud

Este artículo fue sometido a una comprobación de similitud utilizando el software de detección de texto **iThenticate** de Turnitin, a través del servicio de **Similarity Check** de la Crossref.



Proceso de evaluación

Revisión por pares a doble ciego (**Double blind peer review**).

Editor en Jefe

Fredy E. González

Editoras Convidadas

Liliane Gutierrez
Adriana de Bortoli

Publisher

Este artículo ha sido publicado en la revista **Paradigma** vinculada al Centro de Investigaciones Educativas Paradigma (CIEP) del Departamento del Componente Docente de la **Universidad Pedagógica Experimental Libertador** (Núcleo Maracay). La revista **Paradigma** publica artículos de carácter técnico-científico, derivados de estudios e investigaciones que sirvan de apoyo al desarrollo del conocimiento educativo, propiciando el diálogo entre los diferentes campos de la educación. Las ideas expresadas en este artículo son de los autores y no representan necesariamente la opinión del consejo editorial o de la universidad. En Brasil, la revista **Paradigma** obtuvo la calificación **Qualis A1** en la **Evaluación CAPES (2017-2020)**.

